Chapter 3

Due: 11:59pm on Sunday, September 18, 2016

To understand how points are awarded, read the Grading Policy for this assignment.

Introduction to Projectile Motion

Description: Conceptual questions about projectile motion and some easy calculations.

Learning Goal:

To understand the basic concepts of projectile motion.

Projectile motion may seem rather complex at first. However, by breaking it down into components, you will find that it is really no different than the one-dimensional motions that you have already studied.

One of the most often-used techniques in physics is to divide two- and three-dimensional quantities into components. For instance, in projectile motion, a particle has some initial velocity \( \vec{v} \). In general, this velocity can point in any direction on the \( xy \) plane and can have any magnitude. To make a problem more manageable, it is common to break up such a quantity into its \( x \) component \( (v_x) \) and its \( y \) component \( (v_y) \).

Consider a particle with initial velocity \( \vec{v} \) that has magnitude 12.0 m/s and is directed 60.0 degrees above the negative \( x \) axis.

Part A
What is the \( x \) component \( v_x \) of \( \vec{v} \)?

Express your answer in meters per second.

ANSWER:

\[ v_x = -6.00 \text{ m/s} \]

Part B
What is the \( y \) component \( v_y \) of \( \vec{v} \)?

Express your answer in meters per second.

ANSWER:

\[ v_y = 10.4 \text{ m/s} \]

Breaking up the velocities into components is particularly useful when the components do not affect each other. Eventually, you will learn about situations in which the components of velocity do affect one another, but for now you will only be looking at problems where they do not. So, if there is acceleration in the \( x \) direction but not in the \( y \) direction, then the \( x \) component of the velocity will change, but the \( y \) component of the velocity will not.

Part C

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Click on the image below to launch the video: Projectile Motion. Once you have watched the entire video, answer the graded follow-up questions. You can watch the video again at any point.

The motion diagram for a projectile is displayed, as are the motion diagrams for each component. The x-component motion diagram is what you would get if you shone a spotlight down on the particle as it moved and recorded the motion of its shadow. Similarly, if you shone a spotlight to the left and recorded the particle’s shadow, you would get the motion diagram for its y component. How would you describe the two motion diagrams for the components?

ANSWER:

- Both the vertical and horizontal components exhibit motion with constant nonzero acceleration.
- The vertical component exhibits motion with constant nonzero acceleration, whereas the horizontal component exhibits constant-velocity motion.
- The vertical component exhibits constant-velocity motion, whereas the horizontal component exhibits motion with constant nonzero acceleration.
- Both the vertical and horizontal components exhibit motion with constant velocity.

As you can see, the two components of the motion obey their own independent kinematic laws. For the vertical component, there is an acceleration downward with magnitude $g = 10 \text{ m/s}^2$. Thus, you can calculate the vertical position of the particle at any time using the standard kinematic equation $y = y_0 + v_0 t + \frac{1}{2} a t^2$. Similarly, there is no acceleration in the horizontal direction, so the horizontal position of the particle is given by the standard kinematic equation $x = x_0 + v_0 t$.

Part D

How long $t_g$ does it take for the balls to reach the ground? Use $10 \text{ m/s}^2$ for the magnitude of the acceleration due to gravity.

Express your answer in seconds to two significant figures.

**Hint 1. How to approach the problem**

The balls are released from rest at a height of 5.0 m at time $t = 0$ s. Using these numbers and basic kinematics, you can determine the amount of time it takes for the balls to reach the ground.

ANSWER:

$$t_g = 1.0 \text{ s}$$
This situation, which you have dealt with before (motion under the constant acceleration of gravity), is actually a special case of projectile motion. Think of this as projectile motion where the horizontal component of the initial velocity is zero.

**Part E**

Imagine the ball on the left is given a nonzero initial speed in the horizontal direction, while the ball on the right continues to fall with zero initial velocity. What horizontal speed $v_x$ must the ball on the left start with so that it hits the ground at the same position as the ball on the right? Remember that when the two balls are released, they are starting at a horizontal distance of 3.0 m apart.

**Express your answer in meters per second to two significant figures.**

**Hint 1. How to approach the problem**

Recall from Part B that the horizontal component of velocity does not change during projectile motion. Therefore, you need to find the horizontal component of velocity $v_x$ such that, in a time $t_x = 1.0$ s, the ball will move horizontally 3.0 m. You can assume that its initial $x$ coordinate is $x_0 = 0.0$ m.

**ANSWER:**

$v_x = 3.0 \text{ m/s}$

**Exercise 3.8**

**Description:** A remote-controlled car is moving in a vacant parking lot. The velocity of the car as a function of time is given by $\mathbf{v}(t) = (5.00 \text{ m/s} - 0.0180 \text{ m/s}^3 t^2) \mathbf{i} + (2.00 \text{ m/s} + 0.550 \text{ m/s}^2 t) \mathbf{j}$. (a) What is $a_x(t)$ the $x$-component...

A remote-controlled car is moving in a vacant parking lot. The velocity of the car as a function of time is given by

$$\mathbf{v} = [5.00 \text{ m/s} - (0.0180 \text{ m/s}^3 t^2) \mathbf{i} + (2.00 \text{ m/s} + (0.550 \text{ m/s}^2 t) \mathbf{j}].$$

**Part A**

What is $a_x(t)$ the $x$-component of the acceleration of the car as function of time?

**ANSWER:**

- $a_x(t) = (-0.0360 \text{ m/s}^3) t$
- $a_x(t) = (-0.0180 \text{ m/s}^3) t$
- $a_x(t) = (0.0360 \text{ m/s}^3) t$

**Part B**

What is $a_y(t)$ the $y$-component of the acceleration of the car as function of time?

**ANSWER:**

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Part C

What is the magnitude of the velocity of the car at \( t = 7.42 \) s?

Express your answer to three significant figures and include the appropriate units.

ANSWER:

\[
v = \sqrt{(5 - 0.018t^2)^2 + (2 + 0.55t)^2} = 7.28 \frac{m}{s}
\]

Part D

What is the direction (in degrees counterclockwise from +x-axis) of the velocity of the car at \( t = 7.42 \) s?

Express your answer to three significant figures and include the appropriate units.

ANSWER:

\[
\theta = \tan^{-1}\left(\frac{2+0.55t}{5-0.018t^2}\right) \cdot 180 \quad = 56.6^\circ
\]

Part E

What is the magnitude of the acceleration of the car at \( t = 7.42 \) s?

Express your answer to three significant figures and include the appropriate units.

ANSWER:

\[
\alpha = \sqrt{(-0.036t)^2 + 0.55^2} = 0.611 \frac{m}{s^2}
\]

Part F

What is the direction (in degrees counterclockwise from +x-axis) of the acceleration of the car at \( t = 7.42 \) s?

Express your answer to three significant figures and include the appropriate units.

ANSWER:

\[
\theta = 90 + \tan^{-1}\left(\frac{0.036t}{0.55}\right) \cdot 180 = 116^\circ
\]
Exercise 3.14

Description: The froghopper, *Philaenus spumarius*, holds the world record for insect jumps. When leaping at an angle of ## degree(s)## above the horizontal, some of the tiny critters have reached a maximum height of ## cm## above the level ground. (a) What was the...

The froghopper, *Philaenus spumarius*, holds the world record for insect jumps. When leaping at an angle of 58.0 ° above the horizontal, some of the tiny critters have reached a maximum height of 58.7 cm above the level ground.

Part A

What was the takeoff speed for such a leap?

ANSWER:

\[ v = 4.00 \text{ m/s} \]

Part B

What horizontal distance did the froghopper cover for this world-record leap?

ANSWER:

\[ d = 147 \text{ cm} \]

Exercise 3.28

Description: The radius of the earth's orbit around the sun (assumed to be circular) is 1.50 * 10^8 (km), and the earth travels around this orbit in 365 days. (a) What is the magnitude of the orbital velocity of the earth in m/s? (b) What is the radial...

The radius of the earth's orbit around the sun (assumed to be circular) is 1.50 \times 10^8 \text{ km}, and the earth travels around this orbit in 365 days.

Part A

What is the magnitude of the orbital velocity of the earth in m/s?

ANSWER:

\[ 2.97 \times 10^4 \text{ m/s} \]

Part B

What is the radial acceleration of the earth toward the sun?

ANSWER:

\[ 5.91 \times 10^{-3} \text{ m/s}^2 \]
Part C

What is the magnitude of the orbital velocity of the planet Mercury (orbit radius = \(5.79 \times 10^7\) km, orbital period = 88.0 days)?

ANSWER:

\[4.78 \times 10^4\text{ m/s}\]

Part D

What is the radial acceleration of the Mercury?

ANSWER:

\[3.95 \times 10^{-2}\text{ m/s}^2\]

Exercise 3.27

Description: A Ferris wheel with radius 14.0 m is turning about a horizontal axis through its center in \(\omega\). The linear speed of a passenger on the rim is constant and equal to \(v\). (a) What is the magnitude of the passenger's acceleration as she passes...

A Ferris wheel with radius 14.0 m is turning about a horizontal axis through its center in \(\omega\). The linear speed of a passenger on the rim is constant and equal to 7.00 m/s.

Part A

What is the magnitude of the passenger's acceleration as she passes through the lowest point in her circular motion?

Express your answer with the appropriate units.

ANSWER:

\[\alpha = \frac{vv}{14} = 3.50\text{ m/s}^2\]
Part B

What is the direction of the passenger's acceleration as she passes through the lowest point in her circular motion?

ANSWER:

- towards the center
- outwards the center

Part C

What is the magnitude of the passenger's acceleration as she passes through the highest point in her circular motion?

Express your answer with the appropriate units.

ANSWER:

\[ \alpha = \frac{vv}{14} = 3.50 \text{ m/s}^2 \]

Part D

What is the direction of the passenger's acceleration as she passes through the highest point in her circular motion?

ANSWER:

- towards the center
- outwards the center

Part E

How much time does it take the Ferris wheel to make one revolution?

Express your answer with the appropriate units.

ANSWER:

\[ t = \frac{2\pi \cdot 14}{v} = 12.6 \text{s} \]

Exercise 3.32

Description: Two piers, A and B, are located on a river: B is 1500 m downstream from A. Two friends must make round trips from pier A to pier B and return. One rows a boat at a constant speed of 4.00 km/h relative to the water; the other walks on the shore at...
Part A

How much time does it take the walker to make the round trip?

Express your answer using two significant figures.

ANSWER:

\[ t = 45 \text{ min} \]

Part B

How much time does it take the rower to make the round trip?

Express your answer using two significant figures.

ANSWER:

\[ t = 88 \text{ min} \]

Exercise 3.38

Description: An airplane pilot wishes to fly due west. A wind of \( v_w \) is blowing toward the south. (a) If the airspeed of the plane (its speed in still air) is \( v_a \), in which direction should the pilot head? (b) What is the speed of the plane over the ground?

An airplane pilot wishes to fly due west. A wind of 84.0 \( \text{km/h} \) is blowing toward the south.

Part A

If the airspeed of the plane (its speed in still air) is 310.0 \( \text{km/h} \), in which direction should the pilot head?

Express your answer as an angle measured north of west.

ANSWER:
Part B

What is the speed of the plane over the ground?

ANSWER:

\[ v = \sqrt{v_x^2 - v_y^2} = 298 \text{ km/h} \]

Problem 3.48

Description: A movie stuntwoman drops from a helicopter that is 30.0 m above the ground and moving with a constant velocity whose components are 10.0 m/s upward and 15.0 m/s horizontal and toward the south. You can ignore air resistance. (a) What is the...

A movie stuntwoman drops from a helicopter that is 30.0 m above the ground and moving with a constant velocity whose components are 10.0 m/s upward and 15.0 m/s horizontal and toward the south. You can ignore air resistance.

Part A

What is the horizontal distance (relative to the position of the helicopter when she drops) at which the stuntwoman should have placed the foam mats that break her fall?

ANSWER:

\[ x = 55.5 \text{ m} \]

Part B

Draw \( x - t \) graph of her motion.

ANSWER:
Part C

Draw \( y - t \) graph of her motion.

ANSWER:
Part E

Draw $v_x - t$ graph of her motion.

ANSWER:

Draw $v_y - t$ graph of her motion.

ANSWER:
Problem 3.52

Description: As a ship is approaching the dock at 45.0 cm/s, an important piece of landing equipment needs to be thrown to it before it can dock. This equipment is thrown at 15.0 m/s at 60.0° above the horizontal from the top of a tower at the edge of the water, 8.75 m above the ship's deck (the figure).

Part A

For this equipment to land at the front of the ship, at what distance $D$ from the dock should the ship be when the equipment is thrown? Air resistance can be neglected.
Problem 3.58

**Description:** In Canadian football, after a touchdown the team has the opportunity to earn one more point by kicking the ball over the bar between the goal posts. The bar is 10.0 ft above the ground, and the ball is kicked from ground level, 36.0 ft horizontally...

In Canadian football, after a touchdown the team has the opportunity to earn one more point by kicking the ball over the bar between the goal posts. The bar is 10.0 ft above the ground, and the ball is kicked from ground level, 36.0 ft horizontally from the bar in . Football regulations are stated in English units, but convert to SI units for this problem.

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**Part A**

There is a minimum angle above the ground such that if the ball is launched below this angle, it can never clear the bar, no matter how fast it is kicked. What is this angle?

**ANSWER:**

\[ \phi = 15.5 \ ^\circ \]

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**Part B**

If the ball is kicked at 51.0 \(^\circ\) above the horizontal, what must its initial speed be if it is just to clear the bar? Express your answer in m/s.

**ANSWER:**

\[
\begin{align*}
    v_0 &= \frac{10.97}{\cos(\phi)} \sqrt{\frac{9.8}{2 (10.97 \tan(\phi) - 3.048)}} \\
    &= 11.9 \ \text{m/s}
\end{align*}
\]

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**Part C**

\[ D = 25.5 \ \text{m} \]

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**Problem 3.62**

**Description:** A rock is thrown from the roof of a building with a velocity \( v_0 \) at an angle of \( \alpha_0 \) from the horizontal. The building has height \( h \). You can ignore air resistance. (a) Calculate the magnitude of the velocity of the rock just before it strikes the... 

A rock is thrown from the roof of a building with a velocity \( v_0 \) at an angle of \( \alpha_0 \) from the horizontal. The building has height \( h \). You can ignore air resistance.

**Part A**

Calculate the magnitude of the velocity of the rock just before it strikes the ground.

**ANSWER:**

\[
\begin{align*}
    v &= \sqrt{v_0^2 + 2gh} \\
    &= \sqrt{10.97 \cos(\phi) \sqrt{\frac{9.8}{2(10.97 \tan(\phi) - 3.048)}} \cdot 3.6} \quad \text{km/h}
\end{align*}
\]

**Problem 3.71**

**Description:** An airplane pilot sets a compass course due west and maintains an airspeed of \( v \). After flying for a time of \( t \), she finds herself over a town a distance \( x \) west and a distance \( y \) south of her starting point. (a) Find the magnitude of the wind velocity. ...

An airplane pilot sets a compass course due west and maintains an airspeed of 215 \( \text{km/h} \). After flying for a time of 0.490 \( \text{h} \), she finds herself over a town a distance 123 \( \text{km} \) west and a distance 22 \( \text{km} \) south of her starting point.

**Part A**

Find the magnitude of the wind velocity.

**ANSWER:**

\[
\begin{align*}
    v &= \sqrt{\left(\frac{y}{t}\right)^2 + \left(\frac{x - vt}{t}\right)^2} \\
    &= \sqrt{\left(\frac{22\text{ km}}{0.490\text{ h}}\right)^2 + \left(\frac{123\text{ km} - 215\text{ km/h} \cdot 0.490\text{ h}}{0.490\text{ h}}\right)^2} \\
    &= 57.6 \text{ km/h}
\end{align*}
\]

**Part B**

Find the direction of the wind velocity.

**Express your answer as an angle measured south of west**

**ANSWER:**
Part C

If the wind velocity is 45 \( \text{km/h} \) due south, in what direction should the pilot set her course to travel due west? Use the same airspeed of 215 \( \text{km/h} \).

Express your answer as an angle measured north of west

**ANSWER:**

\[
\theta = \frac{\arctan \left( \frac{W}{215-45} \right) \cdot 180}{\pi} = 51.3^\circ \text{ south of west}
\]

\[
\theta = \frac{\arcsin \left( \frac{45}{215} \right) \cdot 180}{\pi} = 12.1^\circ \text{ north of west}
\]