Chapter 3

Motion in Two or Three Dimensions

Chapter 3

Kinematics in Two or Three Dimensions:

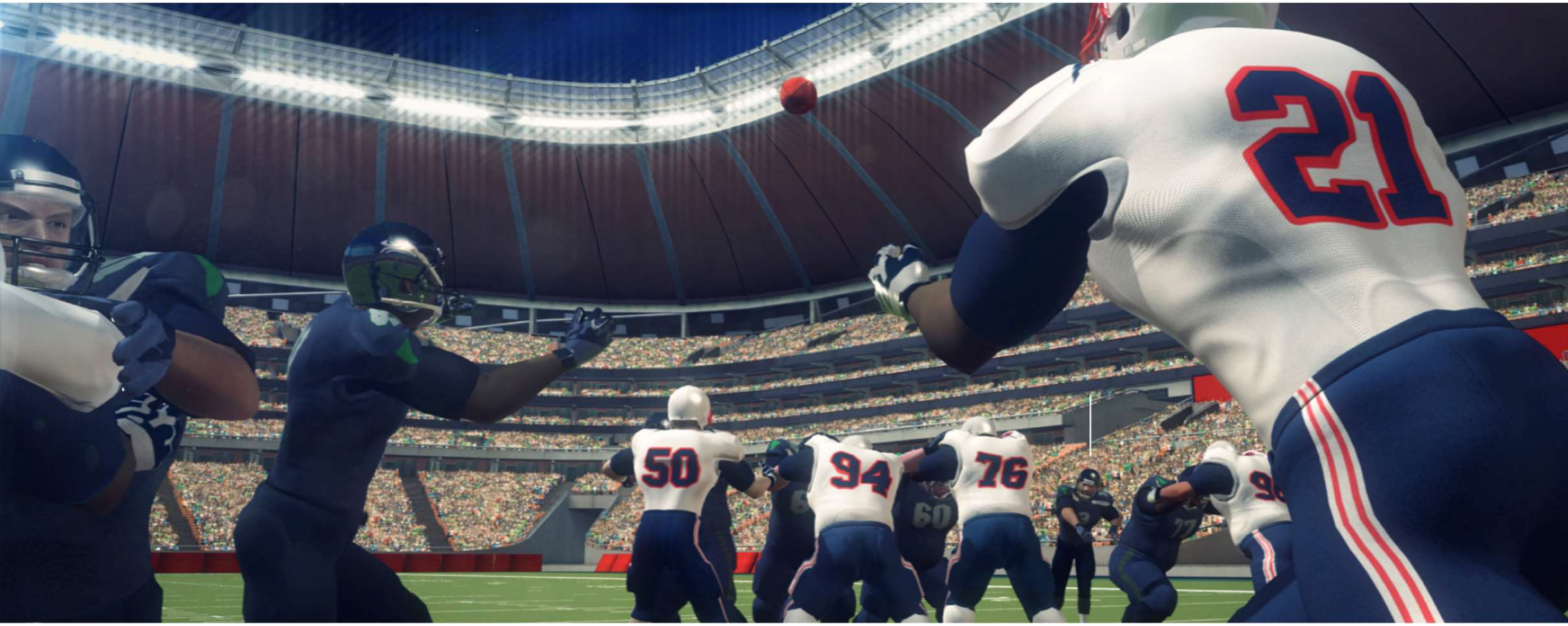
position in 3D, velocity in 3D, acceleration in 3D

Projectile Motion

Uniform Circular Motion

Relative motion

Why do we need to learn projectile motion?



Courtesy: New York Times

Important Equations of Motion

If the acceleration is constant

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

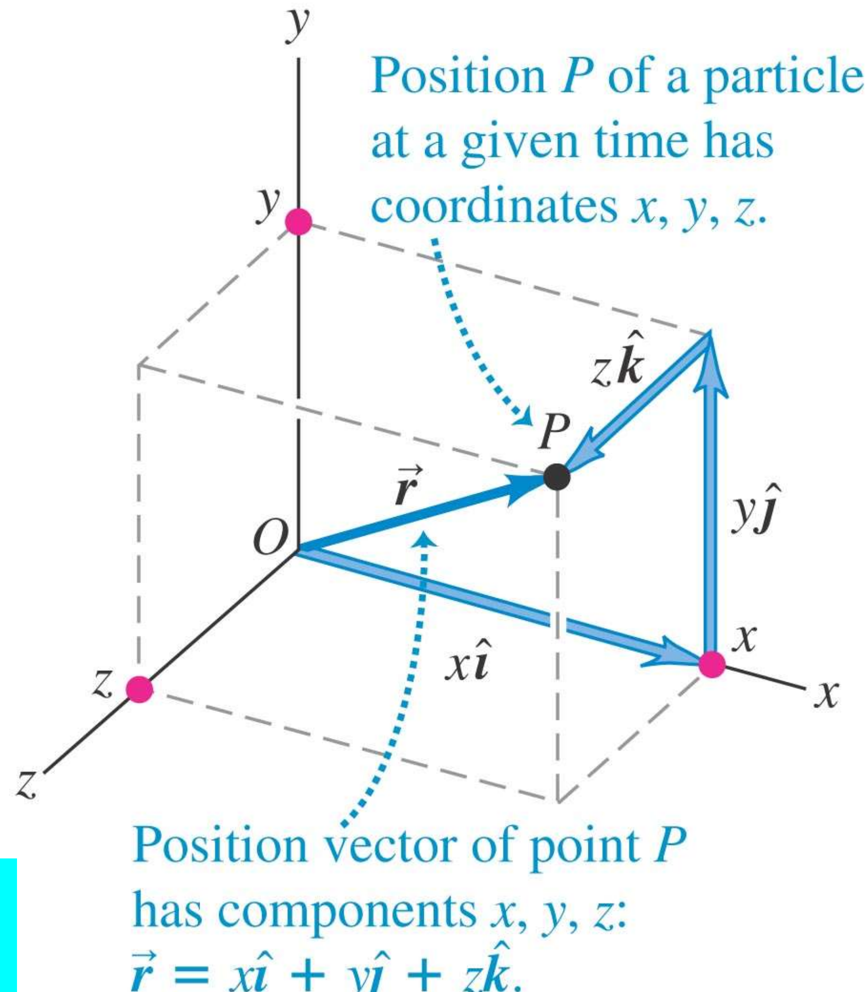
$$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$$

Position in 3 dimensions

Write $\vec{R}$ as our position, relative to the origin in 3 dimensions

$$\vec{R} = X\hat{i} + Y\hat{j} + Z\hat{k}$$

$$\vec{R}(t) = X(t)\hat{i} + Y(t)\hat{j} + Z(t)\hat{k}$$



Velocity in 3 dimensions

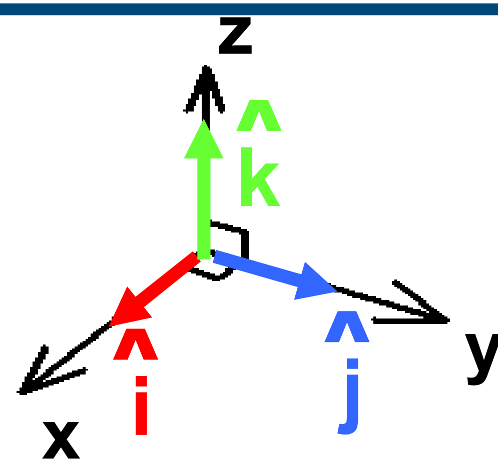
If $\vec{R}$ is the position, then

$$\vec{V} = \frac{d\vec{R}}{dt}$$

$$= \frac{d(X\hat{i} + Y\hat{j} + Z\hat{k})}{dt}$$

$$= \frac{dX}{dt}\hat{i} + \frac{dY}{dt}\hat{j} + \frac{dZ}{dt}\hat{k}$$

$$= V_x\hat{i} + V_y\hat{j} + V_z\hat{k}$$



$$\vec{a} = \frac{d\vec{V}}{dt}$$

$$= \frac{d(V_x\hat{i} + V_y\hat{j} + V_z\hat{k})}{dt}$$

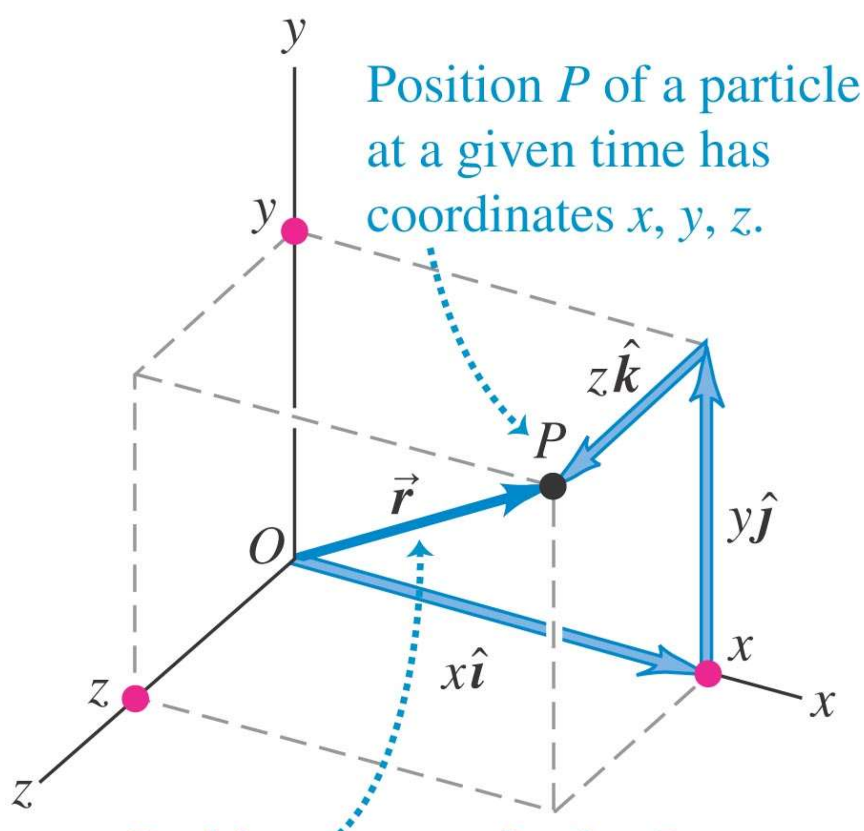
$$= \frac{dV_x}{dt}\hat{i} + \frac{dV_y}{dt}\hat{j} + \frac{dV_z}{dt}\hat{k}$$

$$= a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

Note: We have used $\frac{d\hat{i}}{dt} = 0 = \frac{d\hat{j}}{dt} = \frac{d\hat{k}}{dt}$

Position in 3 dimensions

Write $\vec{R}$ as our position, relative to the origin in 3 dimensions



$$\vec{R} = X\hat{i} + Y\hat{j} + Z\hat{k}$$

$$\vec{R}(t) = X(t)\hat{i} + Y(t)\hat{j} + Z(t)\hat{k}$$

Projectile Motion

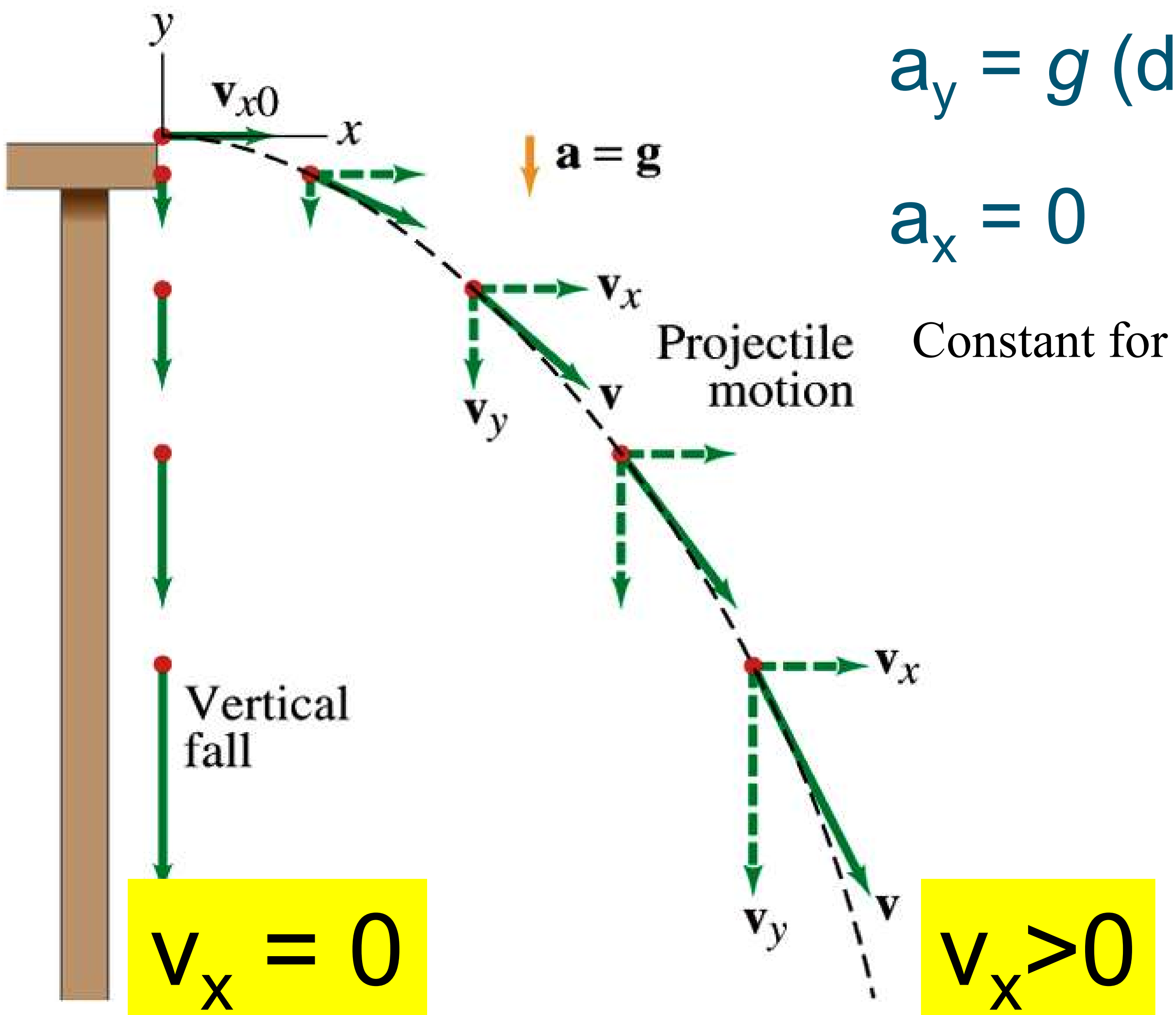
a_y=-g and a_x=0

<u>The horizontal and vertical equations of the motion behave independently</u>

<u>Problem solving:</u>

The trick for all these problems is to break them up into the X and Y directions.

Ball Dropping



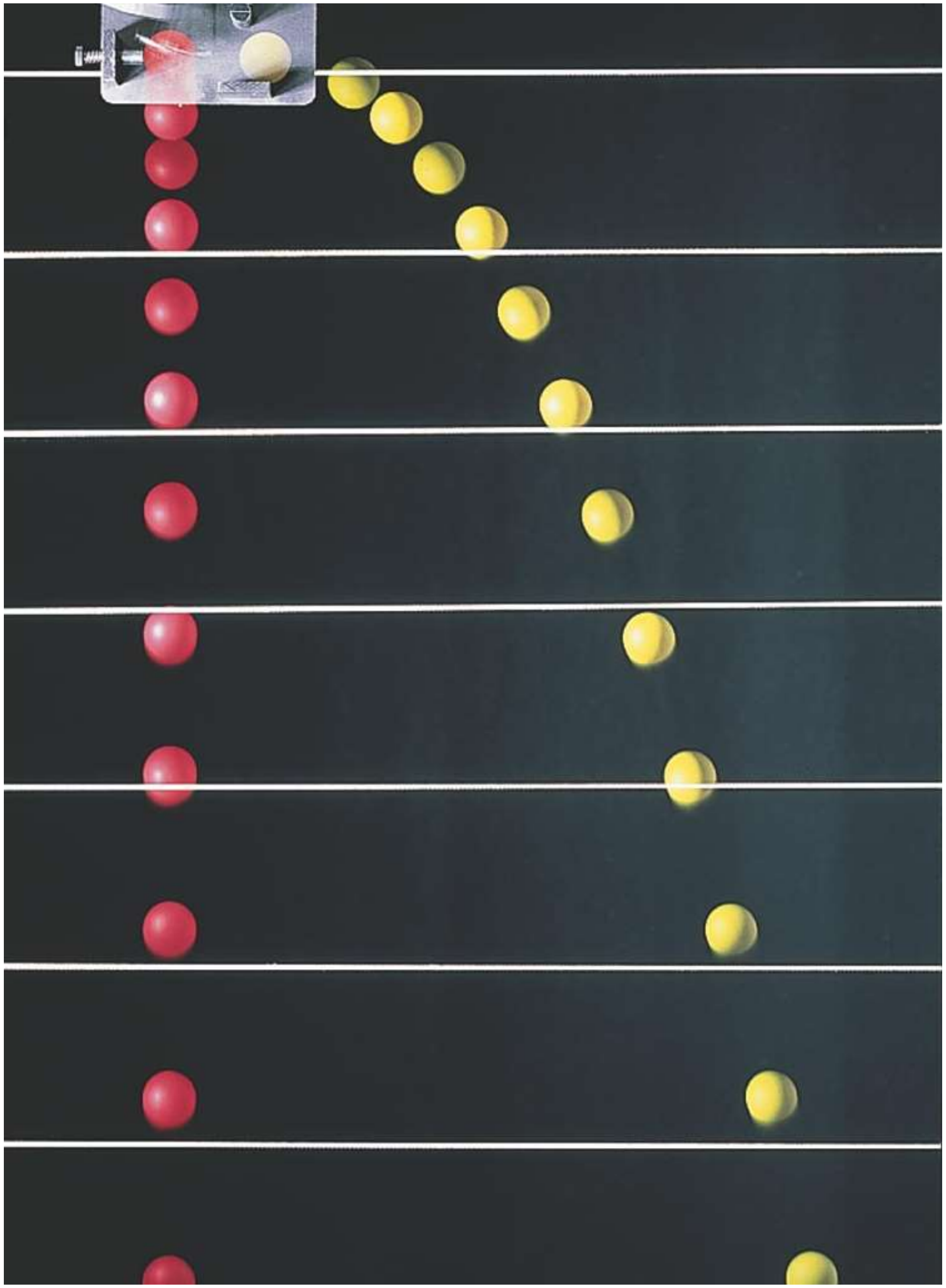
$a_y = g$ (downwards)

$a_x = 0$

Constant for both cases!!!

X and Y motion are separable—Figure 3.16

- The red ball is dropped, and the yellow ball is fired horizontally as it is dropped.
- The strobe marks equal time intervals.



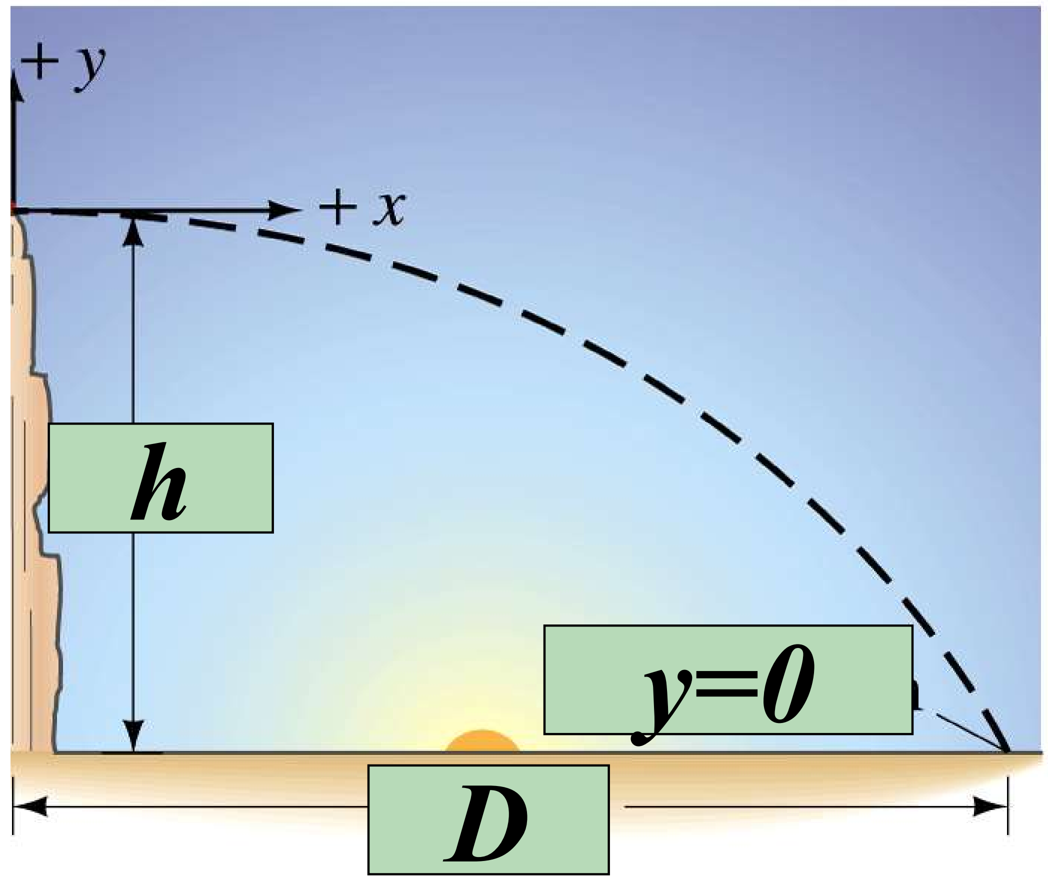
Feedback

The most difficult concept was making sure to remember that the y is independent of the x, therefore the time it takes for a projection to hit the ground is dependent on the height it starts at or reaches.

Rock and Cliff Problem

A ball is thrown horizontally out off a cliff of a height *h* above the ground. The ball hits the ground a distance *D* from the base of the cliff.

Assuming the ball was moving horizontally at the top and ignoring air friction, what was its initial velocity?



Q3.13

An object is dropped from an airplane flying at a constant speed in a straight line at a constant altitude. If there is no air resistance, the falling object will (as seen from the ground)

A. lag behind the airplane, but still move forward.

B. lag behind the airplane and fall straight down.

C. lag behind the airplane and move backward.

D. remain directly under the airplane.

E. move ahead of the airplane.

Rescue Plane

You are the pilot of a rescue plane. Your mission is to drop supplies to isolated mountain climbers on a rocky ridge a height *h* below. If your plane is traveling horizontally with a speed of V_O:

How far in advance of the recipients (horizontal distance) must the goods be dropped?

Rescue Plane cont.

b) Suppose instead, that your plane can't get to that position. Instead you must release the supplies a horizontal distance *D* in advance of the mountain climbers. What *vertical* velocity should you give the supplies so that they arrive precisely at the the climbers position.

c) What is the speed of the supplies as they hit the ground?

Equations of motion under constant acceleration

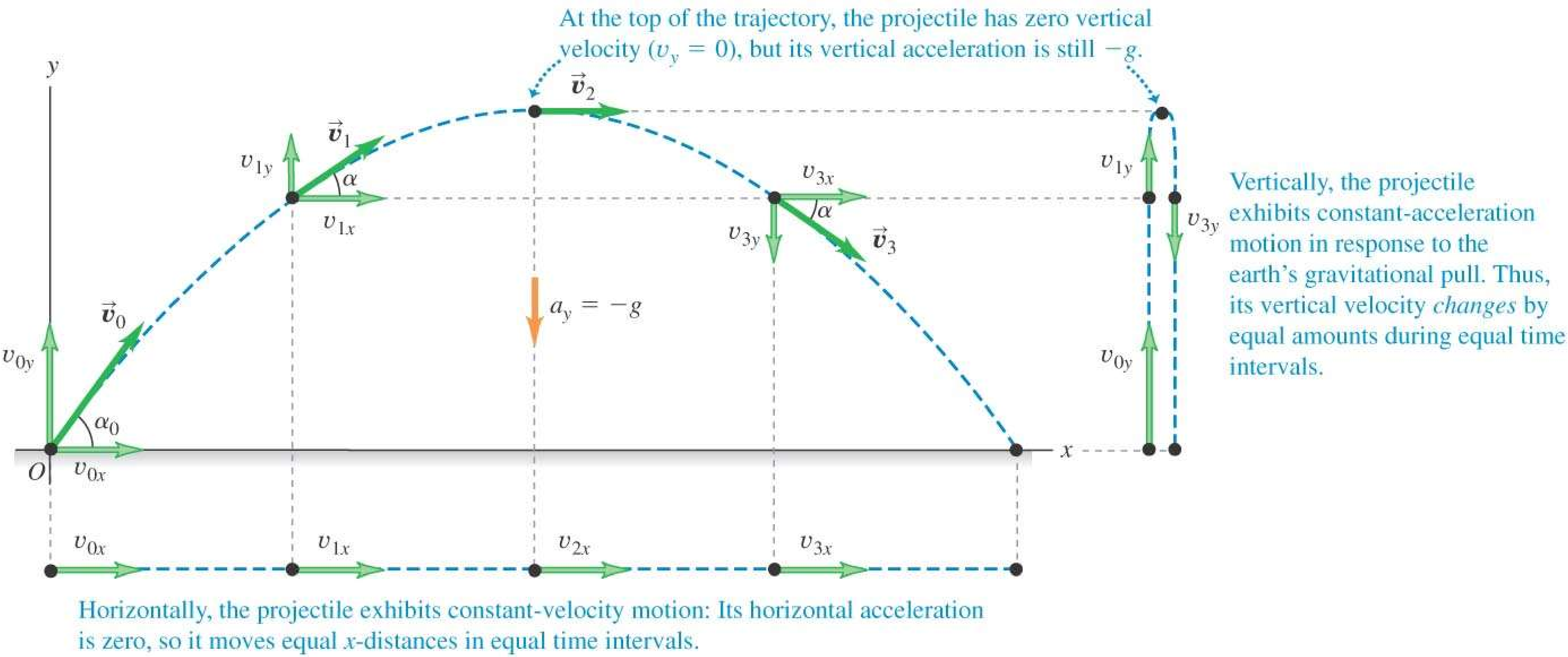
Projectile motion sets $x_o = 0$ and $y_o = 0$ then obtains the specific results shown at right.

$$x = (v_o \cos\alpha_o)t$$

$$y = (v_o \sin\alpha_o)t - 1/2gt^2$$

$$v_x = v_o \cos\alpha_o$$

$$v_y = v_o \sin\alpha_o - gt$$



Firing up in the air at an angle

A ball is fired up with velocity V_o and angle ϑ_o. What is the range?

y-component

$$y - y_0 = 0$$

$$v_{0y} = v_0 \sin\theta$$

$$v_y =$$

$$a_y = -g$$

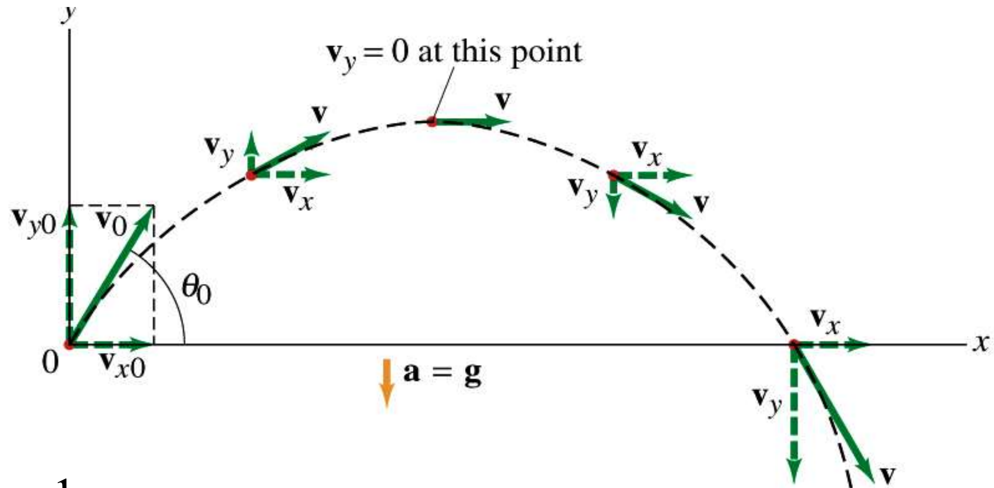
$$t =$$

x-component

$$x - x_0 = R = ?$$

$$v_x = v_0 \cos\theta$$

$$t =$$



Time of flight can be obtained by the x-component

$$x - x_0 = R$$

$$v_x = v_0 \cos\theta$$

$$t = \frac{R}{v_0 \cos\theta}$$

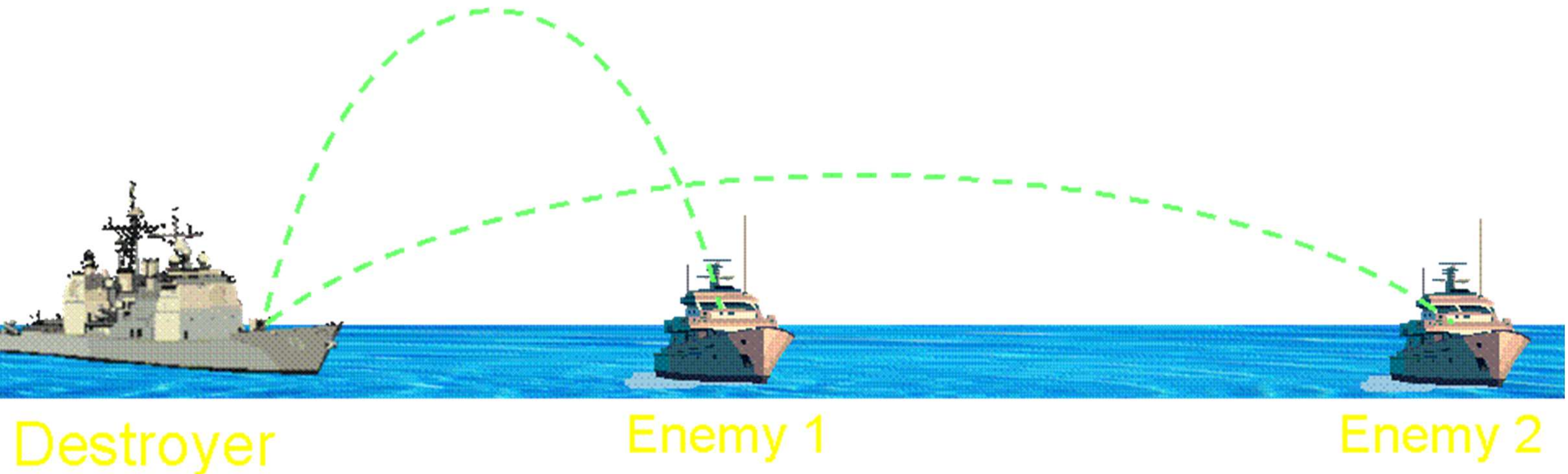
$$y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2$$

$$0 = v_0 \sin\theta \left(\frac{R}{v_0 \cos\theta} \right) - \frac{1}{2} g \left(\frac{R}{v_0 \cos\theta} \right)^2$$

$$\tan\theta = \frac{1}{2} g \frac{R}{v_0{}^2 \cos\theta^2}$$

$$R = 2 \sin\theta \cos\theta \frac{v_0{}^2}{g} = \sin(2\theta) \frac{v_0{}^2}{g}$$

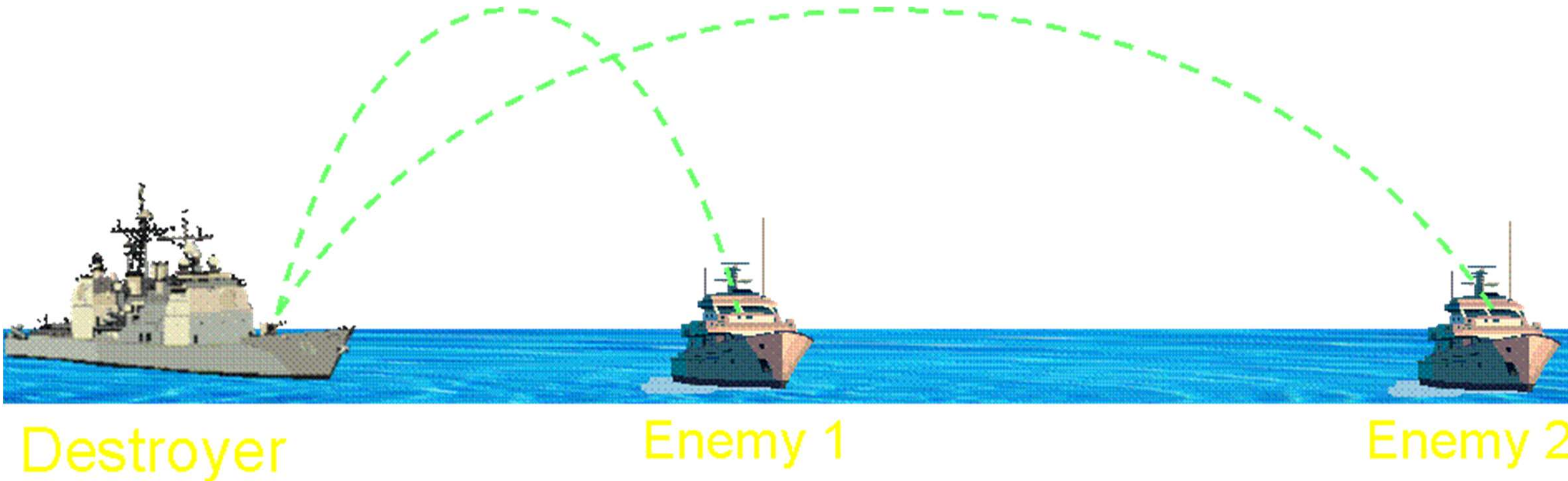
Question



Which ship gets hit first?

a) 1
b) 2
c) Both at same time

Question



Which ship gets hit first?

a) 1
b) 2
c) Both at same time

Kick a football

A football is kicked at angle Θ_0 with respect to the ground with speed V_0. Calculate:

a) The maximum height

b) The velocity at the maximum height

c) The time of travel before the football hits the ground

d) How far away it hits the ground

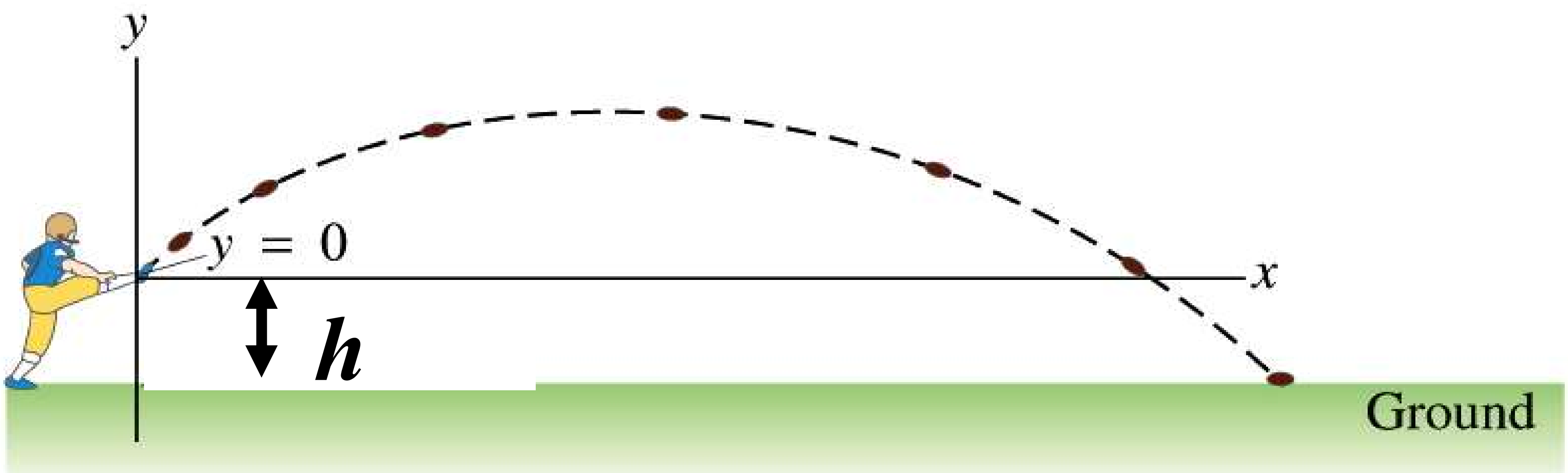
e) What angle maximizes the distance traveled

Assume the ball leaves the foot at ground level and ignore air resistance

Football Punt

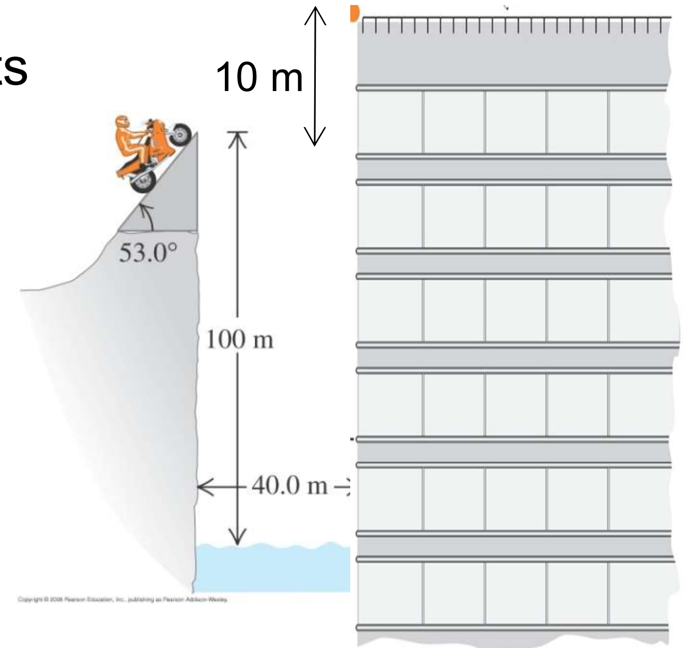
A football is kicked at angle Θ_0 with a velocity V_0. The ball leaves the punters foot *h* meters above the ground.

How far does it travel, in the *X* direction, before it hits the ground?



WHAT INITIAL SPEED DOES HE NEED?

First we break up the problem in x and y components



y-component

$$y - y_0 = h = 10m$$

$$v_{0y} = v_0 \sin\theta$$

$$v_y =$$

$$a_y = -g$$

$$t = \frac{x - x_0}{v_0 \cos\theta}$$

x-component

$$x - x_0 = 40m$$

$$v_x = v_0 \cos\theta$$

$$t =$$

Time of flight can be obtained by the x-component

$$x - x_0 = 40m$$

$$v_x = v_0 \cos\theta$$

$$t = \frac{x - x_0}{v_0 \cos\theta}$$

$$y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2$$

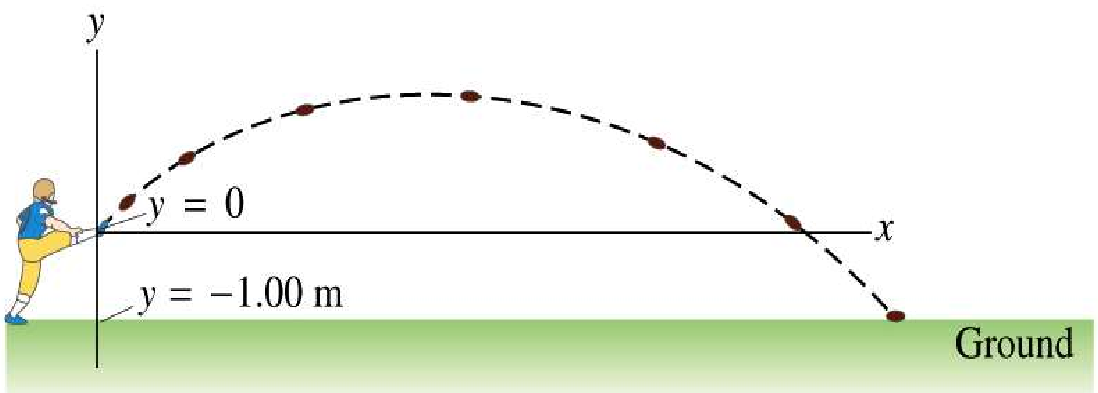
$$h = v_0 \sin\theta \left(\frac{x - x_0}{v_0 \cos\theta} \right) - \frac{1}{2} g \left(\frac{x - x_0}{v_0 \cos\theta} \right)^2$$

$$\tan\theta (x - x_0) - h = \frac{1}{2} g \frac{(x - x_0)^2}{v_0^{\ 2} \cos\theta^2}$$

$$v_0 = \sqrt{\frac{g(x - x_0)^2}{2\cos\theta^2 (\tan\theta (x - x_0) - h)}}$$

Constant Acceleration Review

EACH direction is <u>independent</u>, the only commonality they have is the time



$$x = x_0 + v_{0x} t + \tfrac{1}{2} a_x t^2$$

$$y = y_0 + v_{0y} t + \tfrac{1}{2} a_y t^2$$

$$v_x = v_{0x} + a_x t$$

$$v_y = v_{0y} + a_y t$$

Components of velocity and acceleration

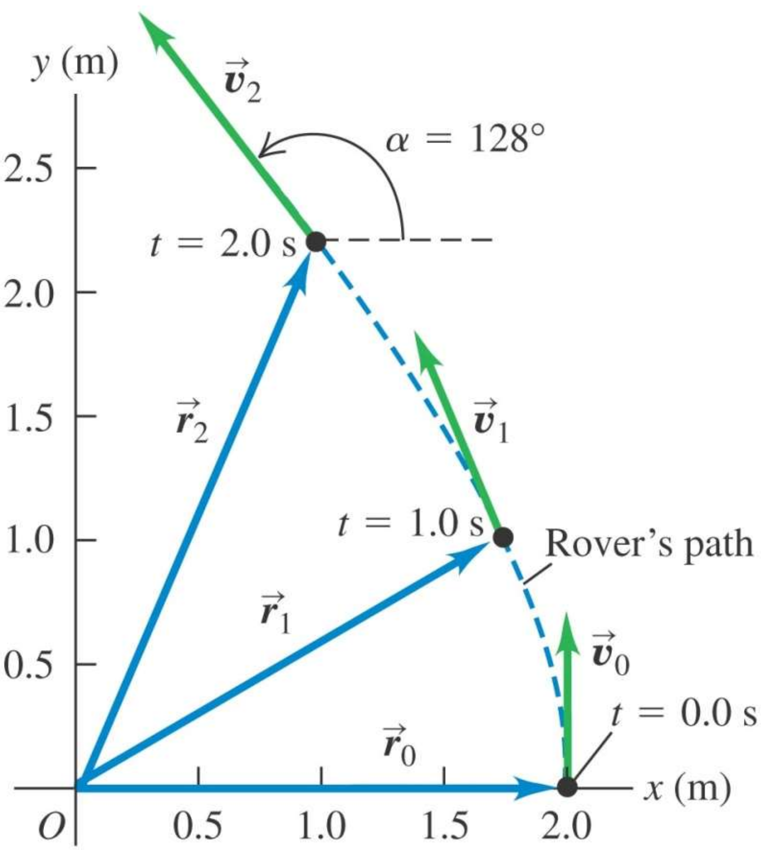
A velocity in the *xy*-plane can be decomposed into separate *x* and *y* components.

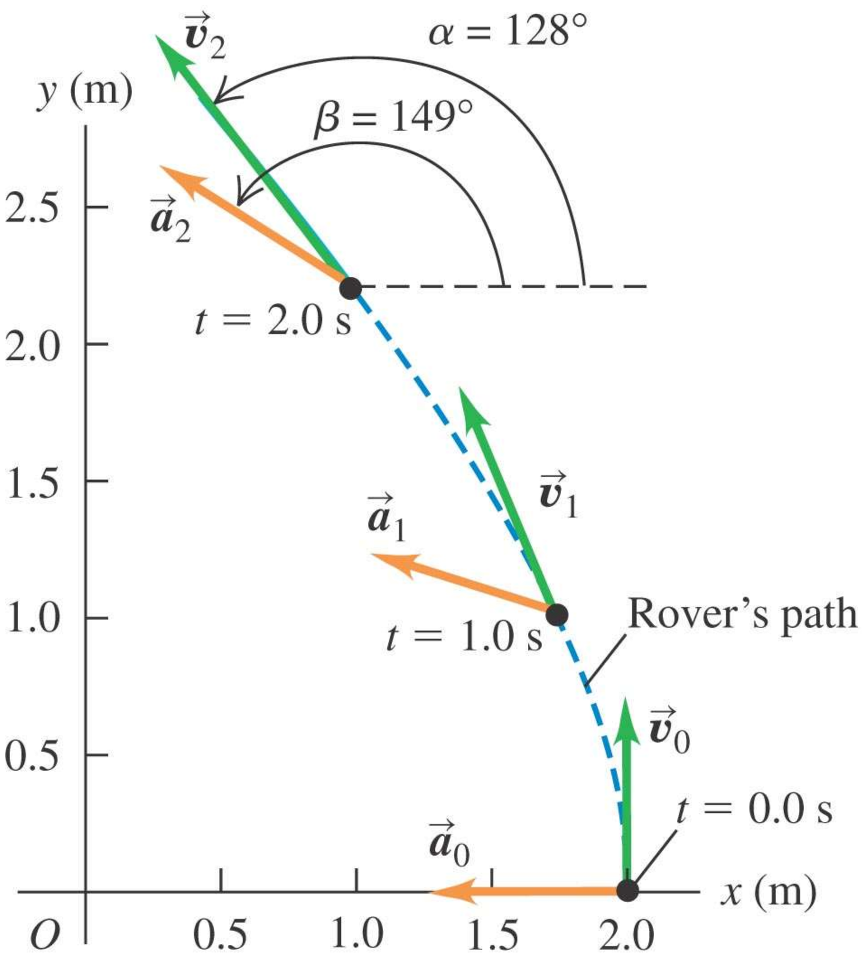
A robotic vehicle, or rover, is exploring the surface of Mars. The landing craft is the origin of coordinates, and the surrounding Martian surface lies in the xy-plane. The rover, which we represent as a point, has x- and y-coordinates that vary with time:

$$x = 2.0\text{ m} - (0.25\text{ m/s}^2)t^2$$
$$y = (1.0\text{ m/s})t + (0.025\text{ m/s}^3)t^3$$

Derivative of each component

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} = (-0.50\text{ m/s}^2)t\hat{\imath} + [1.0\text{ m/s} + (0.075\text{ m/s}^3)t^2]\hat{\jmath}$$
$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} = (-0.50\text{ m/s}^2)\hat{\imath} + (0.15\text{ m/s}^3)t\hat{\jmath}$$



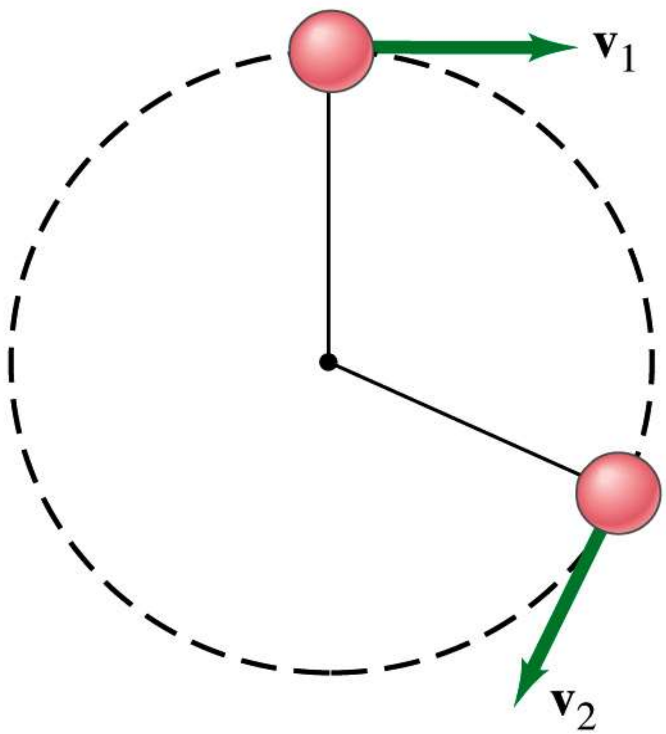


Uniform Circular Motion

Fancy words for moving in a circle with constant *speed*

We see this around us all the time

- Moon around the earth
- Earth around the sun
- Merry-go-rounds

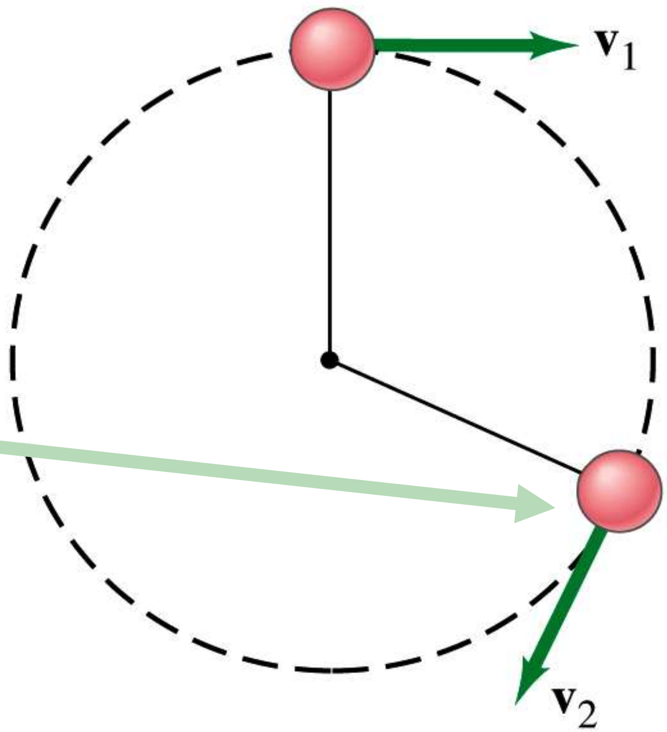


Uniform Circular Motion - Velocity

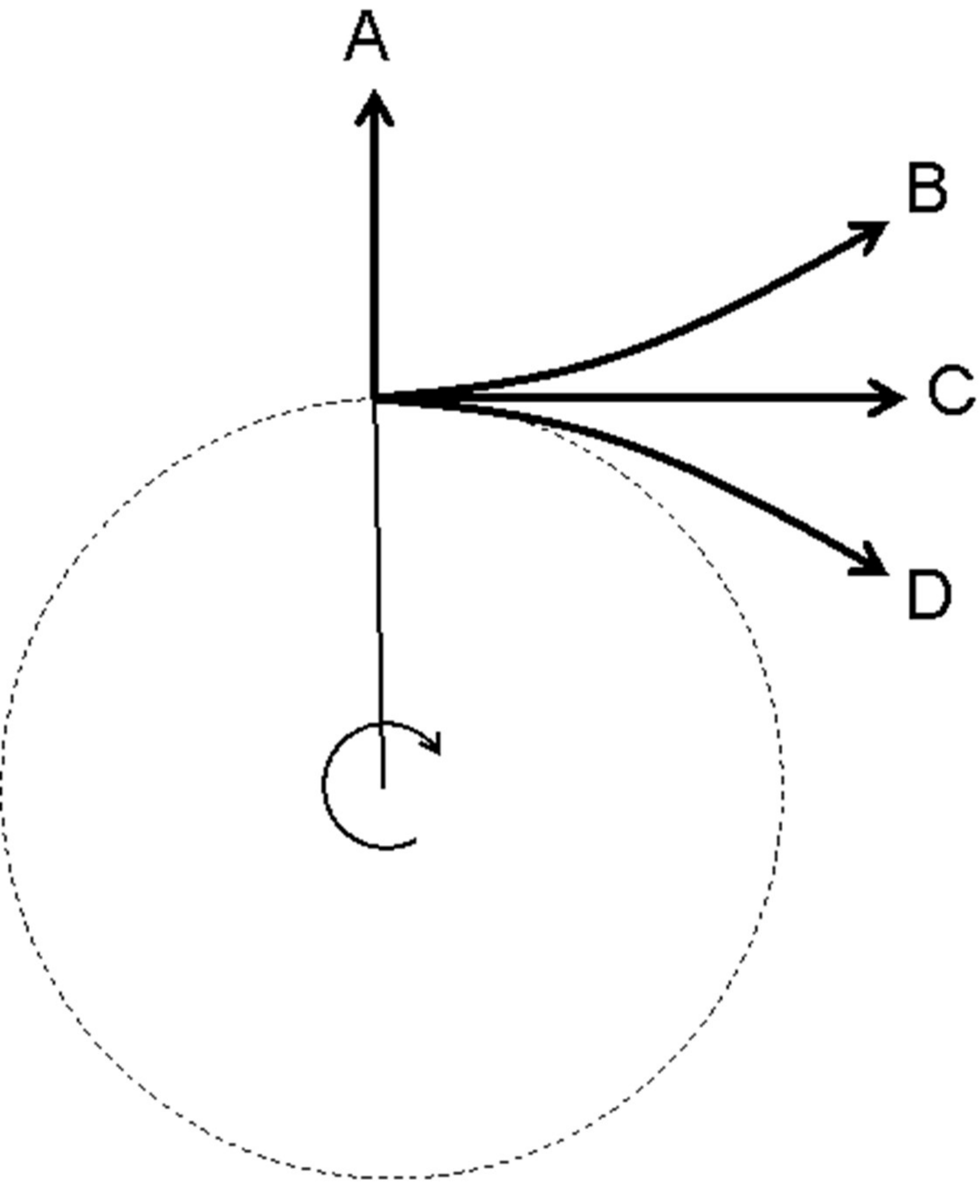
Velocity vector = |V| tangent to the circle

Is this ball accelerating?

- Why?



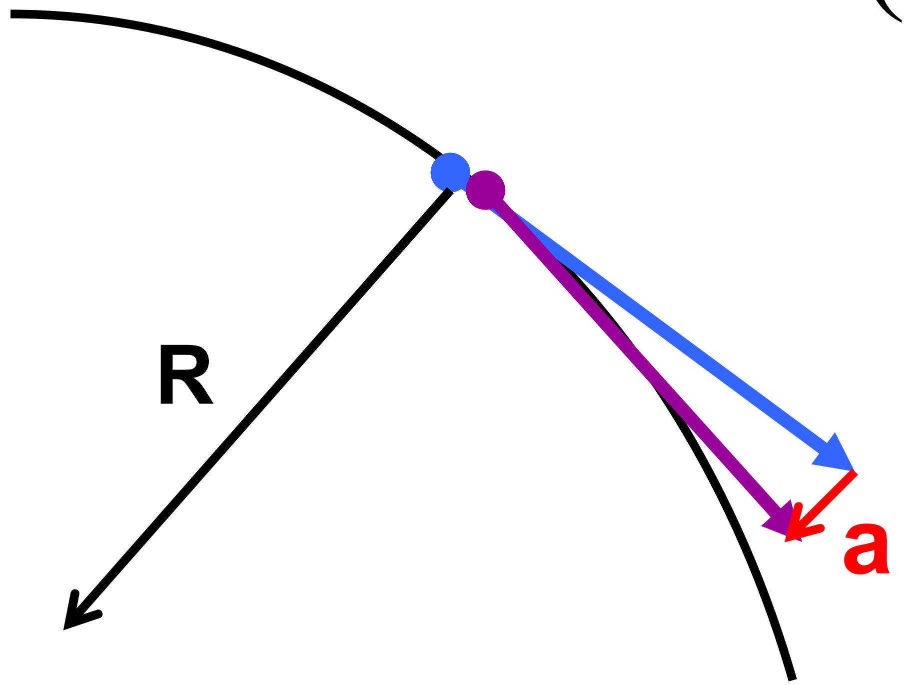
Uniform Circular Motion



A string in a horizontal circle above her head. If the string breaks at the instant shown, which arrow best represents the path the rock will follow?

Centripetal Acceleration

$$\vec{a} = d\vec{v}/dt \approx (\vec{v}_2 - \vec{v}_1)/dt$$



Vector difference V2 - V1 gives the direction of acceleration a

Centripetal Acceleration

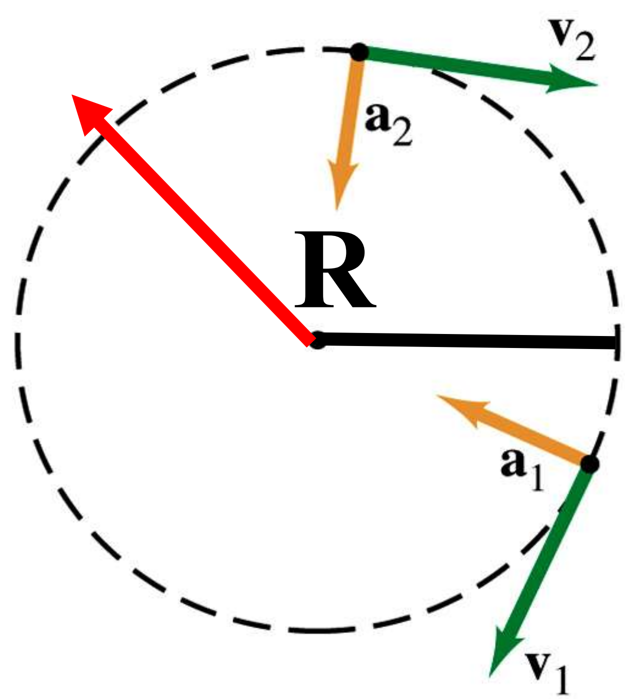
"Center Seeking"

Acceleration vector points *towards the center,* perpendicular to velocity

$$\vec{a} = \frac{v^2}{R}(-\hat{r})$$

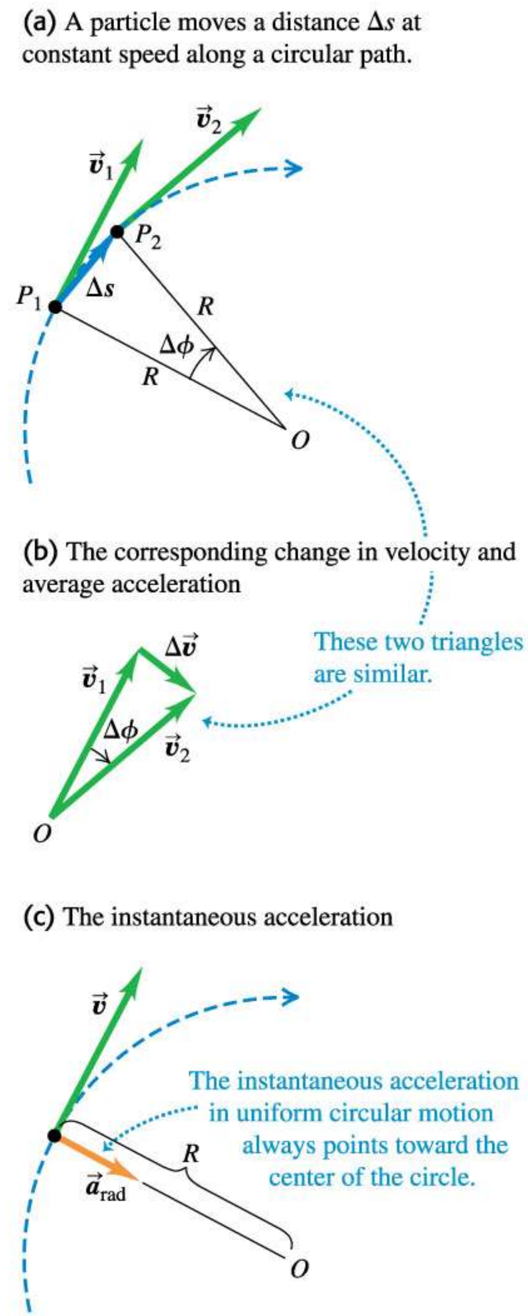
Direction changes, mag same

Unlike projectile motion, where ***a*** has same mag and direction along path



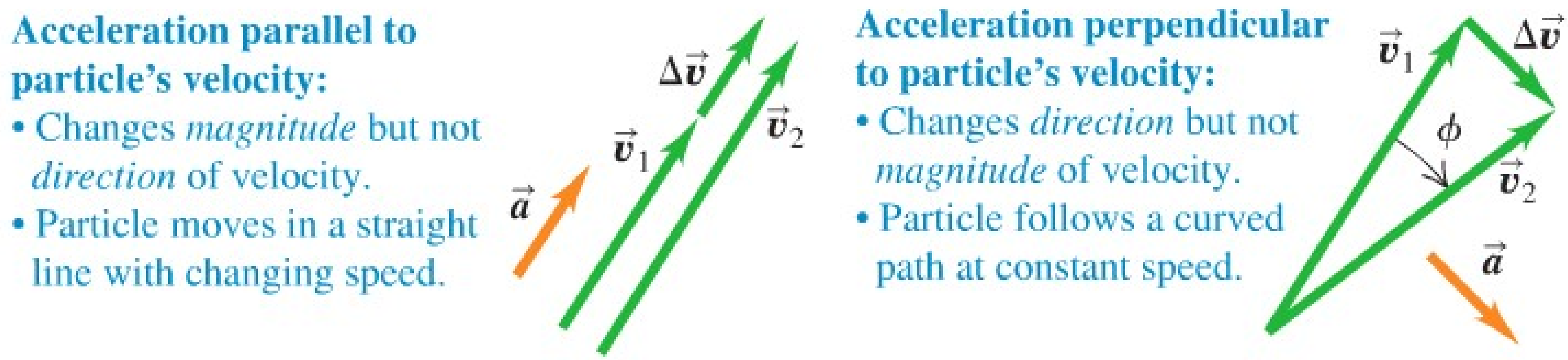
Acceleration for uniform circular motion

- For uniform circular motion, the instantaneous acceleration always points toward the center of the circle and is called the *centripetal acceleration*.
- The magnitude of the acceleration is $a_{rad} = v^2/R$.
- The *period T* is the time for one revolution, and $a_{rad} = 4\pi^2 R/T^2$.



The acceleration vector: graphical interpretation

The acceleration vector can result in a change in either the magnitude OR the direction of the velocity.



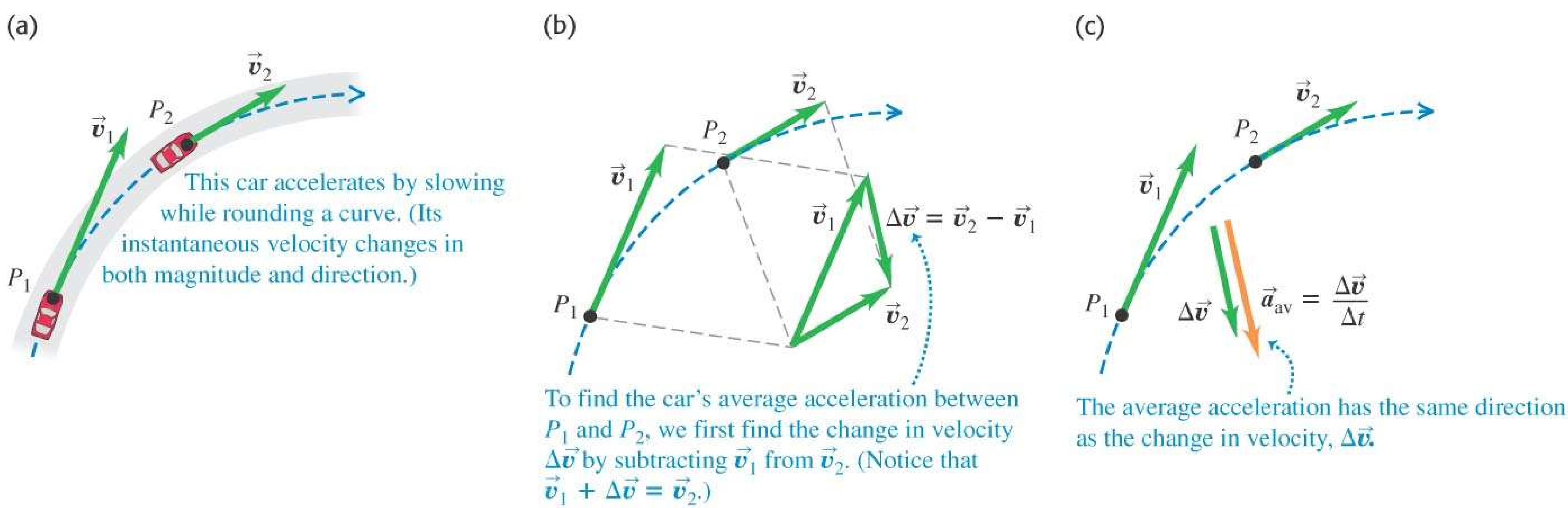
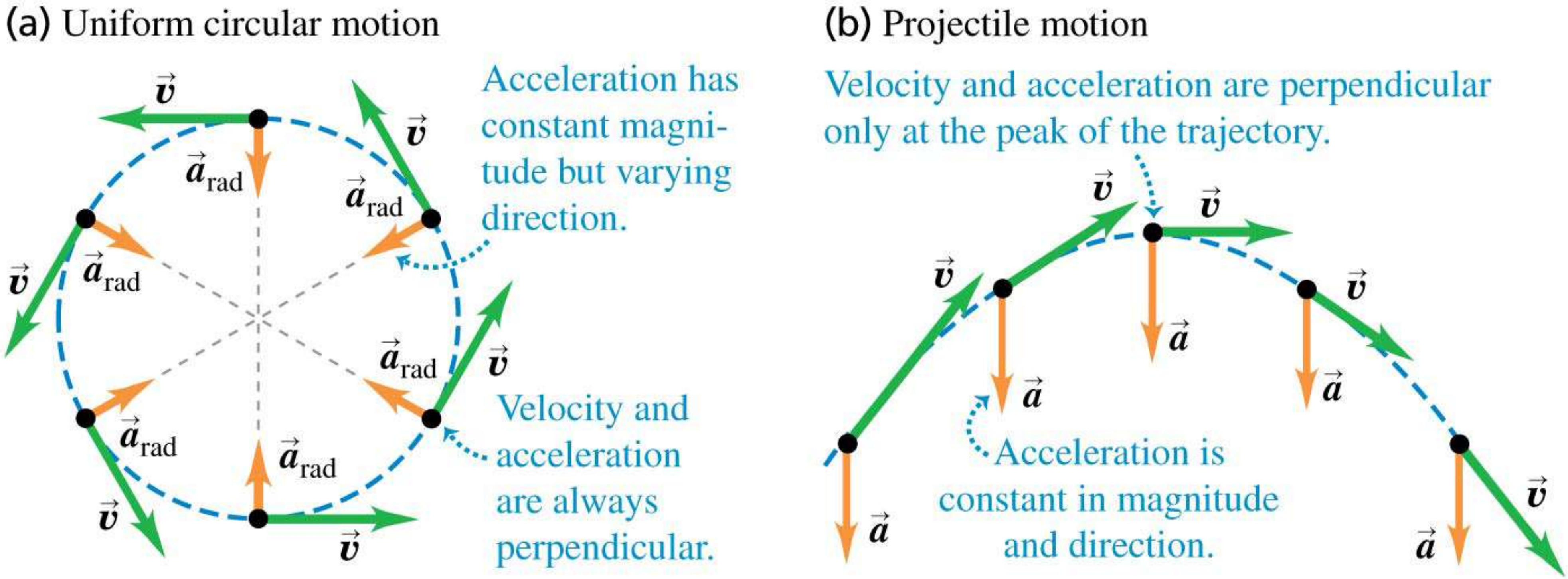
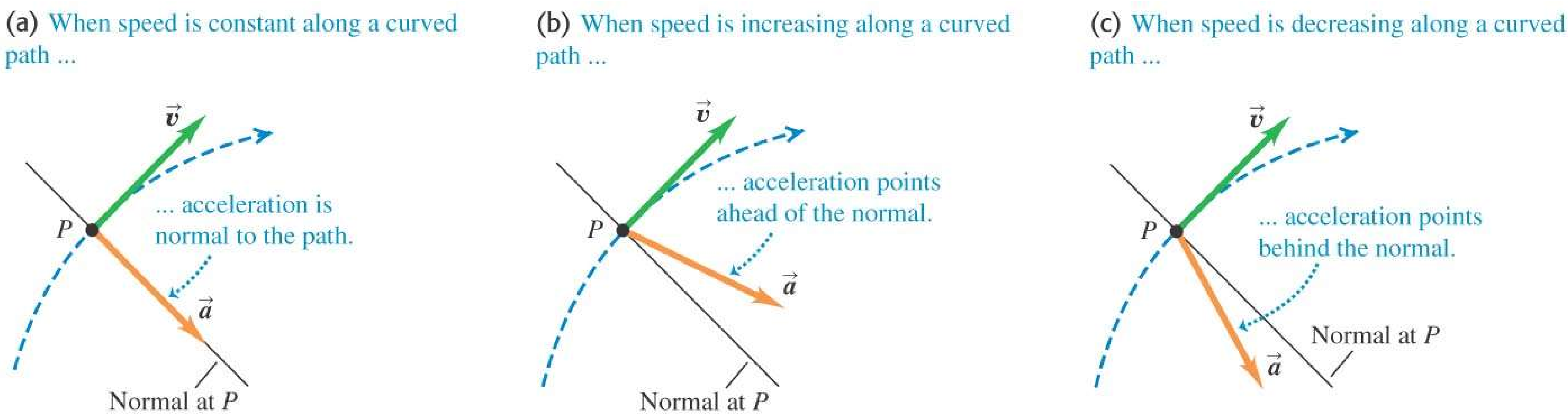


Figure 3.29


(a) Uniform circular motion
Acceleration has constant magnitude but varying direction.
Velocity and acceleration are always perpendicular.
$\vec{v}$
$\vec{a}_{rad}$
(b) Projectile motion
Velocity and acceleration are perpendicular only at the peak of the trajectory.
Acceleration is constant in magnitude and direction.
$\vec{a}$

Different acceleration scenarios

- Notice the acceleration vector change as velocity decreases, remains the same, or increases as one goes around a curve.



Circular Motion: Get the speed!

Speed = distance/time

Distance in 1 revolution divided by the time it takes to go around once

Speed = $2\pi \mathbf{r}/T$

Note: The time to go around once is known as the Period, or T

$$a = v^2/r = (2\pi r/T)^2/r = 4\pi^2 r/T^2$$

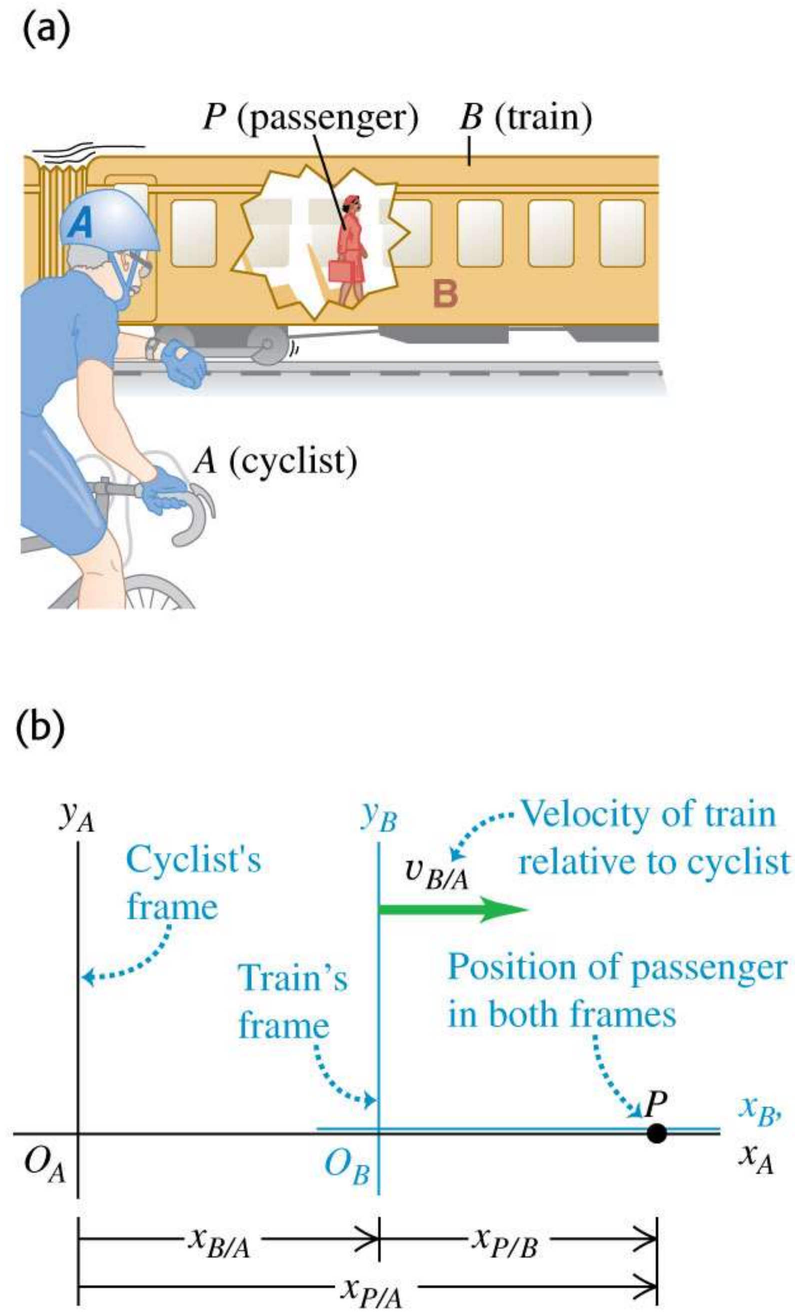
Ball on a String

A ball at the end of a string is revolving uniformly in a horizontal circle (ignore gravity) of radius R. The ball makes N revolutions in a time t.

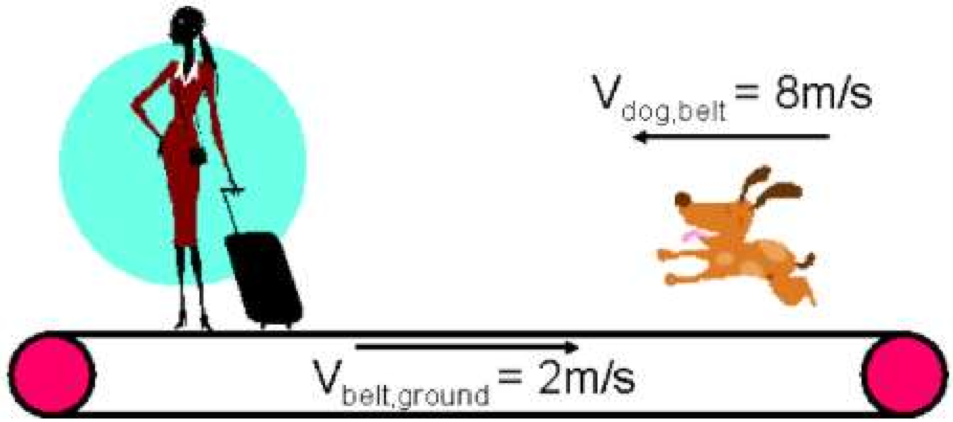
What is the centripetal acceleration?

Relative velocity in one dimension

- If point P is moving relative to reference frame A, we denote the velocity of P relative to frame A as $v_{P/A}$.
- If P is moving relative to frame B and frame B is moving relative to frame A, then the x-velocity of P relative to frame A is $v_{P/A\text{-}x} = v_{P/B\text{-}x} + v_{B/A\text{-}x}$.



A woman stands on a moving sidewalk (conveyor belt) that is moving to the right at a speed of 2 m/s relative to the ground. A dog runs on the belt toward the woman at a speed of 8 m/s relative to the belt.



What is the speed of the dog relative to the ground?

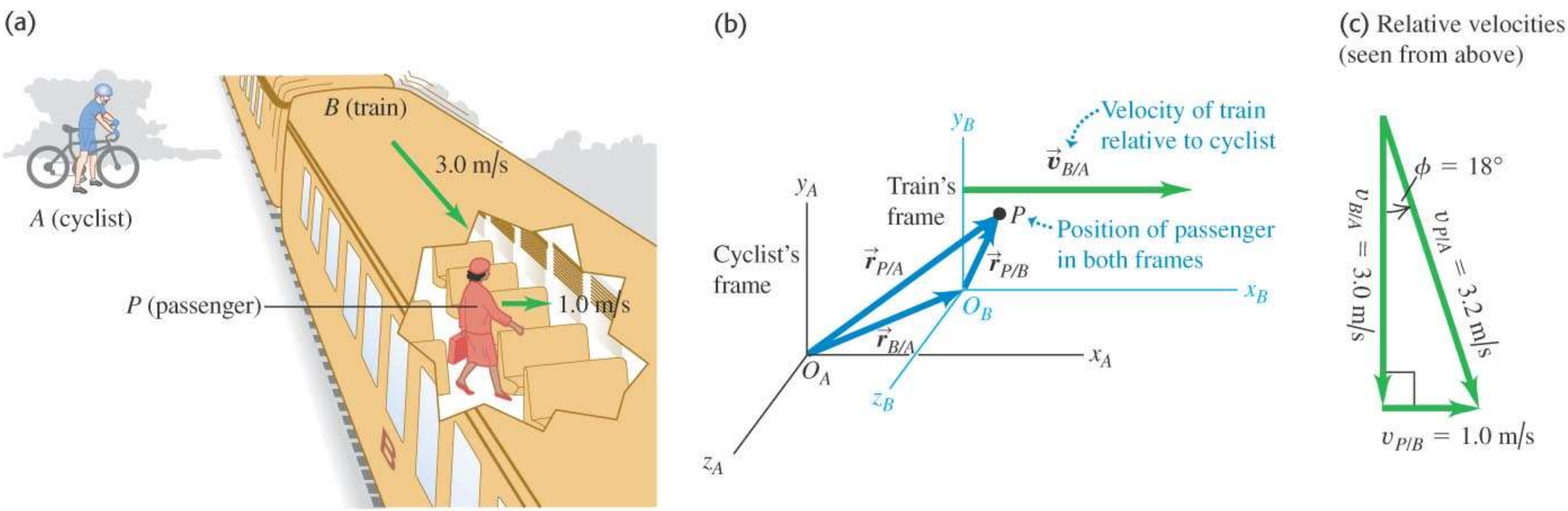
- 6 m/s
- 8 m/s
- 10 m/s

What is the speed of the dog relative to the woman?

- 6 m/s
- 8 m/s
- 10 m/s

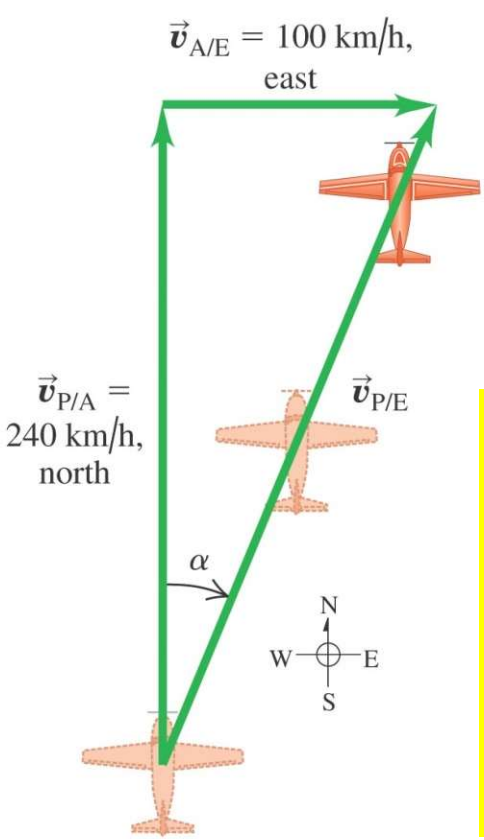
Relative velocity in two or three dimensions

- We extend relative velocity to two or three dimensions by using vector addition to combine velocities.
- In Figure 3.34, a passenger's motion is viewed in the frame of the train and the cyclist.

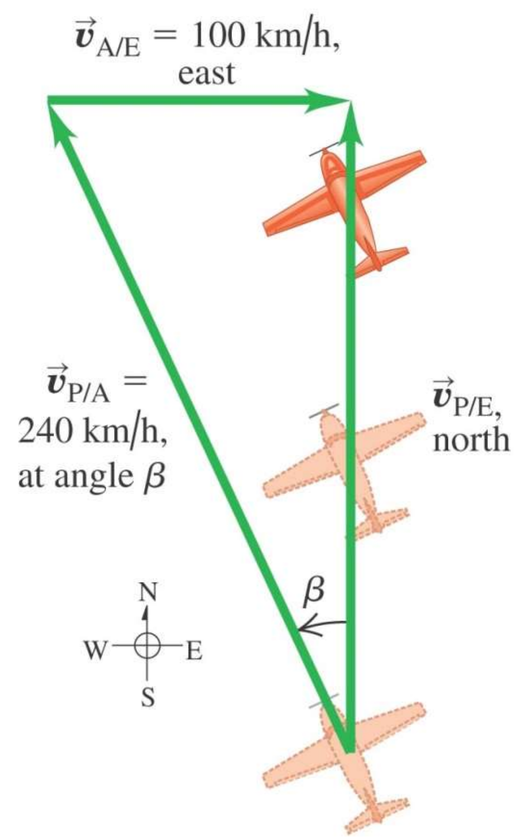


Flying in a crosswind

- A crosswind affects the motion of an airplane.
- An airplane's compass indicates it is headed north, and its airspeed is 240 km/hr. If there is a 100 km/hr wind from west to east, what is the velocity of the airplane relative to earth?

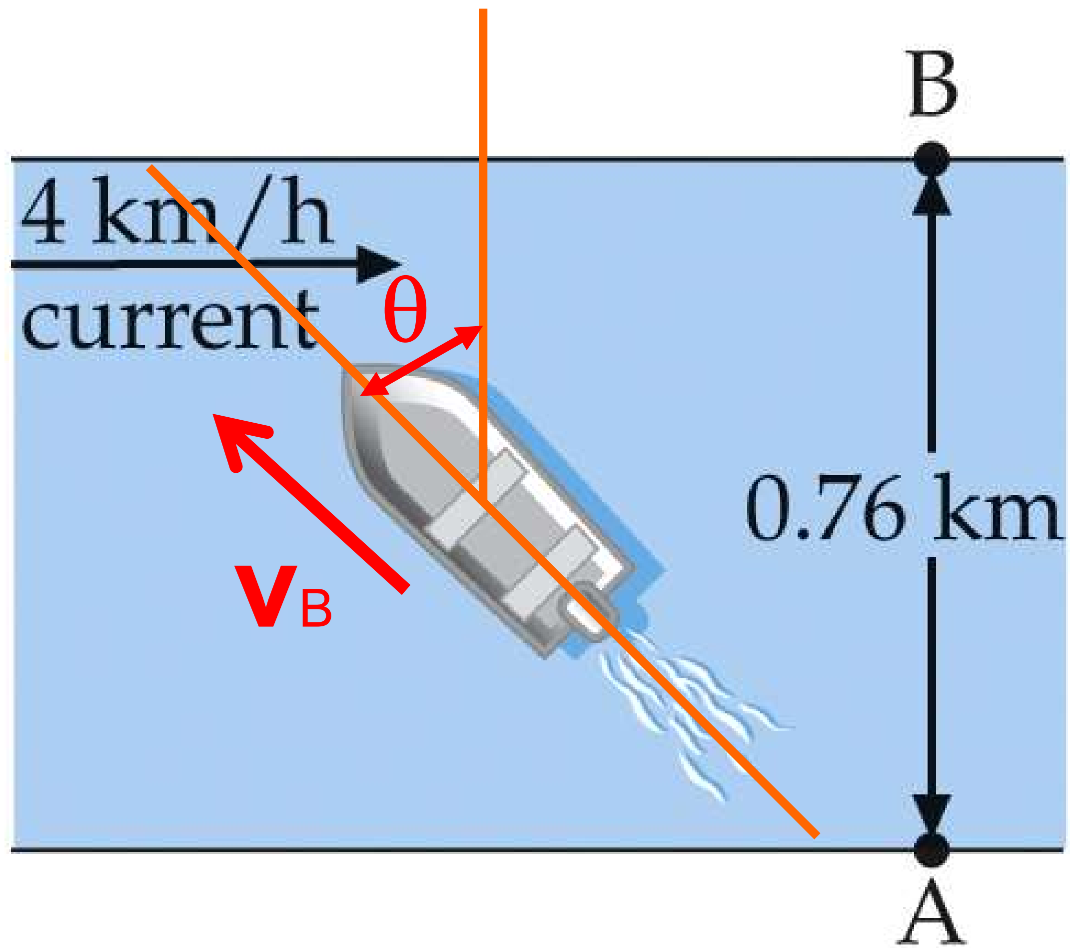


In which direction should the pilot head to travel due north and what will be her velocity relative to earth?



Relative Velocity: Boat on the River

You want to cross the river so that the boat gets exactly from A to B. The river has a current v_C=4 km/h. Your boat's speed in still water is v_B=20km/h?



What is the angle θ you should aim at to do that?

How long would it take?

Heading North

You are the navigator of a flight scheduled to fly from New Orleans due north to St. Louis, a distance of 673 miles. Your instruments tell you that there is a steady wind from the northwest with a speed of 105 mph. The pilot sets the air speed at 575 mph and asks you to find the estimated flying time. What do you tell her?

As a ship is approaching the dock at 45.0 cm/s, an important piece of landing equipment needs to be thrown to it before it can dock. This equipment is thrown at 15.0 m/s at 60.0° above the horizontal from the top of a tower at the edge of the water, 8.75 m above the ship's deck. For this equipment to land at the front of the ship, at what distance from the dock should the ship be when the equipment is thrown? Air resistance can be neglected.

y-component

$$y - y_0 = -h$$

$$v_{0y} = v_0 \sin\theta$$

$$v_y =$$

$$a_y = -g$$

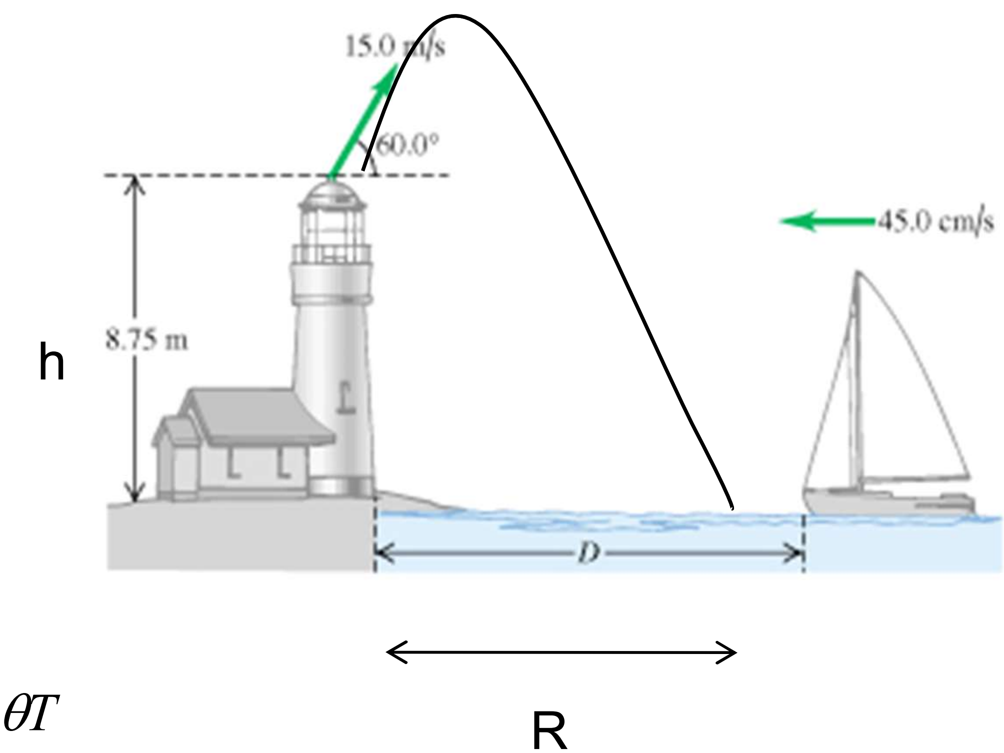
$$t =$$

x-component

$$x - x_0 = R$$

$$v_x = v_0 \cos\theta$$

$$t = T$$



$$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$$

$$-h = v_0 \sin\theta t - \frac{1}{2}gt^2$$

$$t^2 - \frac{2v_0 \sin\theta}{g}t - \frac{2h}{g} = 0$$

$$t = \frac{\frac{2v_0 \sin\theta}{g} \pm \sqrt{\frac{4v^2{}_0 \sin\theta^2}{g^2} + \frac{8h}{g}}}{2} = T$$

$$R = v_0 \cos\theta T$$

$$D = R + v_B T$$