Unwinding Cylinder

**Description:** Using conservation of energy, find the final velocity of a "yo-yo" as it unwinds under the influence of gravity.

A cylinder with moment of inertia $I$ about its center of mass, mass $m$, and radius $r$ has a string wrapped around it which is tied to the ceiling. The cylinder's vertical position as a function of time is $y(t)$.

At time $t = 0$ the cylinder is released from rest at a height $h$ above the ground.

**Part A**

The string constrains the rotational and translational motion of the cylinder. What is the relationship between the angular rotation rate $\omega$ and $v$, the velocity of the center of mass of the cylinder?

Remember that upward motion corresponds to positive linear velocity, and counterclockwise rotation corresponds to positive angular velocity.

Express $\omega$ in terms of $v$ and other given quantities.

**Hint 1. Key to the constrained motion**

Since the cylinder is wrapped about the string, it rolls without slipping with respect to the string. The constraint relationship to assure this kind of motion can be obtained by considering the relative velocity of the point on the cylinder that is in contact with the string with respect to the string—it should be zero.

**Hint 2. Velocity of contact point**

Find a general expression for the velocity of the point of contact.

Express your answer in terms of the velocity of the center of the cylinder, $v = dy(t)/dt$, $r$, and the rotation rate of the cylinder, $\omega$.

**Hint 1. How to approach this question**

First, find the velocity of the contact point relative to the center of the cylinder. Since this ignores the translational motion of the entire cylinder with respect to the ground, you then need to add back in the
cylinder’s center-of-mass velocity to find the velocity of the contact point with respect to the ground.

**ANSWER:**

\[ v_{\text{contact}} = \omega r - v \]

**ANSWER:**

\[ \omega = \frac{v}{r} \]

**Part B**

In similar problems involving rotating bodies, you will often also need the relationship between angular acceleration, \( \alpha \), and linear acceleration, \( a \). Find \( \alpha \) in terms of \( a \) and \( r \).

**ANSWER:**

\[ \alpha = \frac{a}{r} \]

**Part C**

Suppose that at a certain instant the velocity of the cylinder is \( v \). What is its total kinetic energy, \( K_{\text{total}} \), at that instant? Express \( K_{\text{total}} \) in terms of \( m \), \( r \), \( I \), and \( v \).

**Hint 1. Rotational kinetic energy**

Find \( K_{\text{rot}} \), the kinetic energy of rotation of the cylinder.

Express your answer in terms of \( I \) and \( \omega \).

**ANSWER:**

\[ K_{\text{rot}} = \frac{1}{2} I \omega^2 \]

**Hint 2. Rotational kinetic energy in terms of \( v \)**

Now, use the results of Part A to express the rotational kinetic energy \( K_{\text{rot}} \) in terms of \( I \), \( v \), and \( r \).

**ANSWER:**

\[ K_{\text{rot}} = \frac{1}{2} I \left( \frac{v}{r} \right)^2 \]

**Hint 3. Translational kinetic energy**
Find $K_{\text{trans}}$, the translational kinetic energy.

Express your answer in terms of $m$ and $v$.

\[ K_{\text{trans}} = \frac{1}{2}mv^2 \]

ANSWER:

Find $v_f$, the cylinder's vertical velocity when it hits the ground.

Express $v_f$, in terms of $g$, $h$, $I$, $m$, and $r$.

**Hint 1. Initial energy**

Find the cylinder's total mechanical energy when the cylinder is released from rest at height $h$. Take the gravitational potential energy to be zero at $y = 0$.

Give your answer in terms of $m$, $h$, and the magnitude of the acceleration due to gravity, $g$.

\[ E_i = mgh \]

ANSWER:

**Hint 2. Energy conservation**

Apply conservation of energy to this situation, to find the total kinetic energy of the cylinder just before it hits the ground $K_{\text{total}}(y = 0)$.

Your answer should involve $h$, and not $v_f$.

\[ K_{\text{total}}(y=0) = mgh \]

Also accepted: $E_i$

ANSWER:

\[ v_f = -\sqrt{\frac{2gh}{1 + \frac{I}{mr^2}}} \]
Exercise 10.1

Description: Calculate the torque (magnitude and direction) about point O due to the force $\vec{F}$ in each of the cases sketched in the figure. In each case, the force $\vec{F}$ and the rod both lie in the plane of the page, the rod has length 4.00 m, and the force has magnitude 15.0 N.

Calculate the torque (magnitude and direction) about point $O$ due to the force $\vec{F}$ in each of the cases sketched in the figure. In each case, the force $\vec{F}$ and the rod both lie in the plane of the page, the rod has length 4.00 m, and the force has magnitude 15.0 N.

Part A

Calculate the magnitude of the torque in case (a).

ANSWER:

$$|\tau| = F \cdot 4.00 = 60.0 \text{ N} \cdot \text{m}$$

Part B

Find the direction of the torque in case (a).

ANSWER:

- [ ] into the page
- [ ] out of the page
- [ ] to the right
- [ ] upward
- [ ] the torque is zero

Part C

Calculate the magnitude of the torque in case (b)

ANSWER:
Part D

Find the direction of the torque in case (b).

ANSWER:

- into the page
- out of the page
- to the right
- upward
- the torque is zero

Part E

Calculate the magnitude of the torque in case (c)

ANSWER:

\[ |\tau| = F \cdot 4.00 \sin \left(\frac{120}{180}\pi\right) = 52.0 \ \text{N} \cdot \text{m} \]

Part F

Find the direction of the torque in case (c).

ANSWER:

- into the page
- out of the page
- to the right
- upward
- the torque is zero

Part G

Calculate the magnitude of the torque in case (d)

ANSWER:

\[ |\tau| = F \cdot 2.00 \sin \left(\frac{60}{180}\pi\right) = 26.0 \ \text{N} \cdot \text{m} \]
Part H
Find the direction of the torque in case (d).

ANSWER:
- into the page
- out of the page
- to the right
- upward
- the torque is zero

Part I
Calculate the magnitude of the torque in case (e)

ANSWER:
\[ |\tau| = 0 \text{ N}\cdot\text{m} \]

Part J
Find the direction of the torque in case (e).

ANSWER:
- into the page
- out of the page
- to the right
- upward
- the torque is zero

Part K
Calculate the magnitude of the torque in case (f)

ANSWER:
\[ |\tau| = 0 \text{ N}\cdot\text{m} \]

Part L
Find the direction of the torque in case (f).

ANSWER:
Exercise 10.2

Description: (a) Calculate the net torque about point O for the two forces applied as in the figure. The rod and both forces are in the plane of the page. Take positive torques to be counterclockwise.

Part A

Calculate the net torque about point O for the two forces applied as in the figure. The rod and both forces are in the plane of the page. Take positive torques to be counterclockwise.

\[ F_2 = 12.0 \text{ N} \]
\[ F_1 = 8.00 \text{ N} \]

\[ \theta = 30.0^\circ \]

\[ \tau = -28.0 \text{ N} \cdot \text{m} \]

Exercise 10.6

Description: A metal bar is in the xy-plane with one end of the bar at the origin. A force \( \vec{F} = (A \hat{i} + B \hat{j}) \text{ unit} \) is applied to the bar at the point \( x, y \). (a) What is the position vector \( \vec{r} \) for the point where the force is applied?

A metal bar is in the xy-plane with one end of the bar at the origin. A force \( \vec{F} = (6.50 \text{ N} \hat{i} - 2.80 \text{ N} \hat{j}) \) is applied to the bar at the point \( x = 2.23 \text{ m}, y = 3.75 \text{ m} \).

Part A

What is the position vector \( \vec{r} \) for the point where the force is applied?

Enter the \( x \) and \( y \) components of the radius vector separated by a comma.

ANSWER:
Part B

What are the magnitude of the torque with respect to the origin produced by $\vec{F}$?

Express your answer with the appropriate units.

ANSWER:

$\tau = -(xB) + (yA) = 30.6\,\text{N}\cdot\text{m}$

Part C

What are direction of the torque with respect to the origin produced by $\vec{F}$?

ANSWER:

- $+x$-direction
- $+y$-direction
- $+z$-direction
- $-x$-direction
- $-y$-direction
- $-z$-direction

Exercise 10.9

Description: The flywheel of an engine has moment of inertia $I$ about its rotation axis. (a) What constant torque is required to bring it up to an angular speed of 400 rev/min in 8.00 s, starting from rest?

The flywheel of an engine has moment of inertia $2.70\,\text{kg}\cdot\text{m}^2$ about its rotation axis.

Part A

What constant torque is required to bring it up to an angular speed of 400 rev/min in 8.00 s, starting from rest?

Express your answer with the appropriate units.

ANSWER:

$\tau = \frac{I \left( \frac{400 \, \text{rev}}{60 \, \text{sec}} \cdot 2\pi \right)}{8} = 14.1\,\text{N}\cdot\text{m}$

Exercise 10.12
**Description:** A stone is suspended from the free end of a wire that is wrapped around the outer rim of a pulley, similar to what is shown in . The pulley is a uniform disk with mass \( m \) and radius \( r \) and turns on frictionless bearings. You measure that...

A stone is suspended from the free end of a wire that is wrapped around the outer rim of a pulley, similar to what is shown in . The pulley is a uniform disk with mass 10.0 kg and radius 39.0 cm and turns on frictionless bearings. You measure that the stone travels a distance 12.3 m during a time interval of 2.50 s starting from rest.

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**Part A**

Find the mass of the stone.

**Express your answer with the appropriate units.**

**ANSWER:**

\[
m = \frac{\frac{1}{2}m}{\frac{9.80 \Delta t^2}{2s} - 1} = 3.36 \text{ kg}
\]

---

**Part B**

Find the tension in the wire.

**Express your answer with the appropriate units.**

**ANSWER:**

\[
T = \frac{m \Delta s}{\Delta t^2} = 19.7 \text{ N}
\]

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**Exercise 10.20**

**Description:** A string is wrapped several times around the rim of a small hoop with radius 8.00 cm and mass \( m \). The free end of the string is held in place and the hoop is released from rest (the figure ). After the hoop has descended \( s \), calculate (a) the angular...

A string is wrapped several times around the rim of a small hoop with radius 8.00 cm and mass 0.180 kg. The free end of the string is held in place and the hoop is released from rest (the figure ). After the hoop has descended 85.0 cm, calculate
Exercise 10.22

**Description:** A hollow, spherical shell with mass \( m \) rolls without slipping down a slope angled at \( \theta \). (a) Find the acceleration. (b) Find the friction force. (c) Find the minimum coefficient of friction needed to prevent slipping.

A hollow, spherical shell with mass 2.45 kg rolls without slipping down a slope angled at 32.0 \( ^\circ \).

**Part A**

Find the acceleration.

*Take the free fall acceleration to be \( g = 9.80 \) m/s\(^2\)*

\[ \omega = \frac{\sqrt{9.8s}}{0.08} = 36.1 \text{ rad/s} \]

**Part B**

the speed of its center.

\[ v = \sqrt{(9.8s)} = 2.89 \text{ m/s} \]
Part B

Find the friction force.

Take the free fall acceleration to be \( g = 9.80 \ \text{m/s}^2 \)

ANSWER:

\[ \frac{2}{5} m \sin (\theta) = 5.09 \ \text{N} \]

Part C

Find the minimum coefficient of friction needed to prevent slipping.

ANSWER:

\[ \frac{2}{5} \tan (\theta) = 0.250 \]

Exercise 10.29

Description: A playground merry-go-round has radius \( R \) and moment of inertia \( I \) about a vertical axle through its center, and it turns with negligible friction. (a) A child applies an \( F \) force tangentially to the edge of the merry-go-round for \( t \). If the...

A playground merry-go-round has radius 2.90 m and moment of inertia 2100 kg \cdot \text{m}^2 about a vertical axle through its center, and it turns with negligible friction.

Part A

A child applies an 21.0 N force tangentially to the edge of the merry-go-round for 15.0 s. If the merry-go-round is initially at rest, what is its angular speed after this 15.0 s interval?

ANSWER:

\[ \omega = \frac{FR}{I} t = 0.435 \ \text{rad/s} \]

Part B

How much work did the child do on the merry-go-round?

ANSWER:
Part C

What is the average power supplied by the child?

ANSWER:

\[
P = \frac{0.5I \left( \frac{FR}{t} \right)^2}{t} = 13.2 \text{ W}
\]

Exercise 10.31

Description: A m-kg grinding wheel is in the form of a solid cylinder of radius 0.100 m. (a) What constant torque will bring it from rest to an angular speed of 1200 rev/min in 2.5 s? (b) Through what angle has it turned during that time? (c) Use...

A 2.75-kg grinding wheel is in the form of a solid cylinder of radius 0.100 m.

Part A

What constant torque will bring it from rest to an angular speed of 1200 rev/min in 2.5 s?

Express your answer with the appropriate units.

ANSWER:

\[
\tau = \frac{(0.5m \cdot 0.1^2)}{2.5} \left( \frac{1200}{360} \right) \frac{\pi}{2} = 0.69 \text{ N} \cdot \text{m}
\]

Also accepted: \( \frac{(0.5m \cdot 0.1^2)}{2.5} \left( \frac{1200}{360} \right) \frac{\pi}{2} = 0.691 \text{ N} \cdot \text{m} \), \( \frac{(0.5m \cdot 0.1^2)}{2.5} \left( \frac{1200}{360} \right) \frac{\pi}{2} = 0.69 \text{ N} \cdot \text{m} \)

Part B

Through what angle has it turned during that time?

ANSWER:

\[
\phi = 160 \text{ rad}
\]

Also accepted: 157, 160

Part C

Use equation \( W = \tau \phi (\theta_2 - \theta_1) = \tau \Delta \theta \) to calculate the work done by the torque.

Express your answer with the appropriate units.
ANSWER:

\[ W = \frac{600 \cdot 2.5}{60} \frac{(0.5m \cdot 0.1^2)}{(2\pi)} \left( \frac{1200 \pi}{30} \right) = 110 \text{ J} \]

Also accepted: \[ \frac{600 \cdot 2.5}{60} \frac{(0.5m \cdot 0.1^2)}{(2\pi)} \left( \frac{1200 \pi}{30} \right) = 109 \text{ J}, \quad \frac{600 \cdot 2.5}{60} \frac{(0.5m \cdot 0.1^2)}{(2\pi)} \left( \frac{1200 \pi}{30} \right) = 110 \text{ J} \]

---

**Part D**

What is the grinding wheel's kinetic energy when it is rotating at 1200 rev/min?

Express your answer with the appropriate units.

ANSWER:

\[ K = 0.5 \left( \frac{0.5m \cdot 0.1^2}{30} \right) \left( \frac{1200 \pi}{30} \right)^2 = 110 \text{ J} \]

Also accepted: \[ 0.5 \left( \frac{0.5m \cdot 0.1^2}{30} \right) \left( \frac{1200 \pi}{30} \right)^2 = 109 \text{ J}, \quad 0.5 \left( \frac{0.5m \cdot 0.1^2}{30} \right) \left( \frac{1200 \pi}{30} \right)^2 = 110 \text{ J} \]

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**Part E**

Compare your answer in part (D) to the result in part (C).

ANSWER:

- The results are the same.
- The results are not the same.

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**Exercise 10.35**

**Description:** A x rock has a horizontal velocity of magnitude 12.0 m/s when it is at point P in the figure. (a) At this instant, what is the magnitude of its angular momentum relative to point O? (b) What is the direction of the angular momentum in part (A)? (c)...  

A 2.60 kg rock has a horizontal velocity of magnitude 12.0 m/s when it is at point \( P \) in the figure.
Part A

At this instant, what is the magnitude of its angular momentum relative to point \( O \)?

**ANSWER:**

\[
L = x \cdot 12.8 \sin \left( \frac{36.9 \pi}{180} \right) = 150 \text{ kg} \cdot \text{m}^2/\text{s}
\]

Part B

What is the direction of the angular momentum in part (A)?

**ANSWER:**

- into the page
- out of the page

Part C

If the only force acting on the rock is its weight, what is the magnitude of the rate of change of its angular momentum at this instant?

**ANSWER:**

\[
\left| \frac{dL}{dt} \right| = x \cdot 9.8 \cdot 8 \sin \left( \frac{(90 - 36.9) \pi}{180} \right) = 163 \text{ kg} \cdot \text{m}^2/\text{s}^2
\]

Part D

What is the direction of the rate in part (C)?

**ANSWER:**
Exercise 10.36

Description: A woman with mass 50 kg is standing on the rim of a large disk that is rotating at 0.80 rev/s about an axis through its center. The disk has mass \( m \) and radius 4.0 m. (a) Calculate the magnitude of the total angular momentum of the...

A woman with mass 50 kg is standing on the rim of a large disk that is rotating at 0.80 rev/s about an axis through its center. The disk has mass 130 kg and radius 4.0 m.

Part A

Calculate the magnitude of the total angular momentum of the woman–disk system. (Assume that you can treat the woman as a point.)

Express your answer to two significant figures and include the appropriate units.

ANSWER:

\[
L = \left( \frac{m}{2} + 50 \right) \cdot 4^2 (0.8 \cdot 2\pi) = 9200 \text{ kg} \cdot \text{m}^2 / \text{s}
\]

Also accepted: \( \left( \frac{m}{2} + 50 \right) \cdot 4^2 (0.8 \cdot 2\pi) = 9250 \text{ kg} \cdot \text{m}^2 / \text{s} \), \( \left( \frac{m}{2} + 50 \right) \cdot 4^2 (0.8 \cdot 2\pi) = 9200 \text{ kg} \cdot \text{m}^2 / \text{s} \)

Exercise 10.41

Description: Under some circumstances, a star can collapse into an extremely dense object made mostly of neutrons and called a neutron star. The density of a neutron star is roughly \( 10^{14} \) times as great as that of ordinary solid matter. Suppose we represent the...

Under some circumstances, a star can collapse into an extremely dense object made mostly of neutrons and called a neutron star. The density of a neutron star is roughly \( 10^{14} \) times as great as that of ordinary solid matter. Suppose we represent the star as a uniform, solid, rigid sphere, both before and after the collapse. The star's initial radius was \( 6.0 \times 10^5 \text{ km} \) (comparable to our sun); its final radius is 17 km.

Part A

If the original star rotated once in 27 days, find the angular speed of the neutron star.

Express your answer using two significant figures.

ANSWER:

\[
\omega_2 = \frac{2\pi}{t \cdot 86400} \left( \frac{r_1}{r_2} \right)^2 = 3400 \text{ rad/s}
\]
Exercise 10.42

Description: A diver comes off a board with arms straight up and legs straight down, giving her a moment of inertia about her rotation axis of 18 ( kg) * m^2. She then tucks into a small ball, decreasing this moment of inertia to 3.6 ( kg) * m^2. While tucked,...

A diver comes off a board with arms straight up and legs straight down, giving her a moment of inertia about her rotation axis of 18 kg * m^2. She then tucks into a small ball, decreasing this moment of inertia to 3.6 kg * m^2. While tucked, she makes two complete revolutions in 1.3 s.

Part A

If she hadn't tucked at all, how many revolutions would she have made in the 1.5 s from board to water?

Express your answer using two significant figures.

ANSWER:

\[
\frac{t_2}{t_1} = 0.46 \text{ rev}
\]

Problem 10.62

Description: A block with mass m=## kg slides down a surface inclined 36.9 degree(s) to the horizontal (the figure ). The coefficient of kinetic friction is ##. A string attached to the block is wrapped around a flywheel on a fixed axis at O. The flywheel has...

A block with mass \( m = 5.00 \text{ kg} \) slides down a surface inclined 36.9° to the horizontal (the figure). The coefficient of kinetic friction is 0.27. A string attached to the block is wrapped around a flywheel on a fixed axis at \( O \). The flywheel has mass 25.0 kg and moment of inertia 0.500 kg * m^2 with respect to the axis of rotation. The string pulls without slipping at a perpendicular distance of 0.200 m from that axis.

Part A

What is the acceleration of the block down the plane?

ANSWER:

\[
a = \frac{9.8 \left( 0.6 - 0.8 \right)}{1 + \frac{0.5}{250^2}} = 1.08 \text{ m/s}^2
\]
Part B

What is the tension in the string?

ANSWER:

\[ T = \frac{9.8(0.6 - 0.8 h)}{1 + \frac{0.8 h}{R}} \cdot 0.5 = 13.4 \text{ N} \]

Problem 10.68

Description: A thin-walled, hollow spherical shell of mass \( m \) and radius \( r \) starts from rest and rolls without slipping down the track shown in the figure. Points A and B are on a circular part of the track having radius \( R \). The diameter of the shell is very...

A thin-walled, hollow spherical shell of mass \( m \) and radius \( r \) starts from rest and rolls without slipping down the track shown in the figure. Points \( A \) and \( B \) are on a circular part of the track having radius \( R \). The diameter of the shell is very small compared to \( h_0 \) and \( R \), and the work done by the rolling friction is negligible.

Part A

What is the minimum height \( h_0 \) for which this shell will make a complete loop-the-loop on the circular part of the track?

Express your answer in terms of the variables \( m, R \), and appropriate constants.

ANSWER:

\[ h_0 = \frac{17}{6} R \]

Part B

How hard does the track push on the shell at point \( B \), which is at the same level as the center of the circle?

Express your answer in terms of the variables \( m, R \), and appropriate constants.

ANSWER:
Part C

Suppose that the track had no friction and the shell was released from the same height \( h_0 \) you found in part (a). Would it make a complete loop-the-loop?

ANSWER:

- yes
- no

Part D

In part (c), how hard does the track push on the shell at point \( A \), the top of the circle?

Express your answer in terms of the variables \( m \), \( R \), and appropriate constants.

ANSWER:

\[ |\vec{N}| = \frac{2}{3}mg \]

Part E

How hard did the track push on the shell at point \( A \) in part (a)?

Express your answer in terms of the variables \( m \), \( R \), and appropriate constants.

ANSWER:

\[ |\vec{N}| = 0 \]

Problem 10.70 - Copy

Description: A solid, uniform ball rolls without slipping up a hill, as shown in the figure. At the top of the hill, it is moving horizontally, and then it goes over the vertical cliff. (a) How far from the foot of the cliff does the ball land? (b) How fast is...

A solid, uniform ball rolls without slipping up a hill, as shown in the figure. At the top of the hill, it is moving horizontally, and then it goes over the vertical cliff.
Part A

How far from the foot of the cliff does the ball land?

ANSWER:

\[ l = 36.5 \text{ m} \]

Part B

How fast is it moving just before it lands?

ANSWER:

\[ v = 28.0 \text{ m/s} \]

Problem 10.85

Description: A m_bird bird is flying horizontally at v, not paying much attention, when it suddenly flies into a stationary vertical bar, hitting it h below the top (the figure). The bar is uniform, l long, has a mass of m, and is hinged at its base. The...

A 550.0 g bird is flying horizontally at 2.40 m/s, not paying much attention, when it suddenly flies into a stationary vertical bar, hitting it 25.0 cm below the top (the figure). The bar is uniform, 0.740 m long, has a mass of 1.30 kg, and is hinged at its base. The collision stuns the bird so that it just drops to the ground afterward (but soon recovers to fly happily away).
Part A

What is the angular velocity of the bar just after it is hit by the bird?

ANSWER:

\[ \omega = \frac{3m_{\text{bird}}v(l - h)}{ml^2} = 2.73 \text{ rad/s} \]

Part B

What is the angular velocity of the bar just as it reaches the ground?

ANSWER:

\[ \omega = \sqrt{\left(\frac{3m_{\text{bird}}v(l - h)}{ml^2}\right)^2 + \frac{3g}{l}} = 6.87 \text{ rad/s} \]