it is essential to deal with the same set of particles throughout the time interval $t_a$ to $t_b$: we must keep track of all the particles that were originally in the system. Consequently, the mass of the system cannot change during the time of interest.

**Example 1.11 Mass Flow and Momentum**

A spacecraft moves through space with constant velocity $v$. The spacecraft encounters a stream of dust particles which embed themselves in it at rate $dm/dt$. The dust has velocity $u$ just before it hits. At time $t$ the total mass of the spacecraft is $M(t)$. The problem is to find the external force $F$ necessary to keep the spacecraft moving uniformly. (In practice, $F$ would most likely come from the spacecraft's own rocket engines. For simplicity, we can visualize the source $F$ to be completely external—an invisible hand, so to speak.)

Let us focus on the short time interval between $t$ and $t + \Delta t$. The drawings below show the system at the beginning and end of the interval.

Let $\Delta m$ denote the mass added to the satellite during $\Delta t$. The system consists of $M(t)$ and $\Delta m$. The initial momentum is

$$P(t) = M(t)v + (\Delta m)u.$$ 

The final momentum is

$$P(t + \Delta t) = M(t)v + (\Delta m)v.$$ 

The change in momentum is

$$\Delta P = P(t + \Delta t) - P(t) = (v - u) \Delta m.$$
To analyze the motion of the rocket in detail, we must equate the external force on the system, \( F \), with the rate of change of momentum, \( \frac{dP}{dt} \). Consider the rocket at time \( t \). Between \( t \) and \( t + \Delta t \) a mass of fuel \( \Delta m \) is burned and expelled as gas with velocity \( u \) relative to the rocket. The exhaust velocity \( u \) is determined by the nature of the propellants, the throttling of the engine, etc., but it is independent of the velocity of the rocket.

The sketches below show the system at time \( t \) and at time \( t + \Delta t \).

The system consists of \( \Delta m \) plus the remaining mass of the rocket \( M \). Hence the total mass is \( M + \Delta m \).

The velocity of the rocket at time \( t \) is \( v(t) \), and at \( t + \Delta t \), it is \( v + \Delta v \). The initial momentum is

\[
P(t) = (M + \Delta m)v
\]

and the final momentum is

\[
P(t + \Delta t) = M(v + \Delta v) + \Delta m(v + \Delta v + u).
\]

The change in momentum is

\[
\Delta P = P(t + \Delta t) - P(t) = M \Delta v + (\Delta m)u.
\]

Therefore,

\[
\frac{dP}{dt} = \lim_{\Delta t \to 0} \frac{\Delta P}{\Delta t} = M \frac{dv}{dt} + u \frac{dm}{dt}.
\]

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Note that we have defined \( u \) to be positive in the direction of \( v \). In most rocket applications, \( u \) is negative, opposite to \( v \). It is inconvenient to have both \( m \) and \( M \) in the equation. \( \frac{dm}{dt} \) is
Δm to be added in time Δt

System boundary; mass of system = M(t) + Δm

Time t

F

v

M(t)

System boundary; mass of system = M(t) + Δm

Time t + Δt
\[ \Delta m \text{ to be added in time } \Delta t \]

**System boundary:**

- Mass of system at time \( t \): \( M(t) \)
- Mass of system at time \( t + \Delta t \): \( M(t) + \Delta m \)

\[ u - v \]
Time $t$

LAUNCH FRAME

Time $t + \Delta t$
Time $t$  

**CO-MOVING FRAME**  

Time $t + \Delta t$