PHYSICS 218 Exam 3
Spring, 2014

Wednesday, April 16, 2014

Please read the information on the cover page BUT DO NOT OPEN the exam until instructed to do so!

Name:______________________
Signature:____________________
Student ID:__________________
E-mail:______________________
Section Number: _____________

Rules of the exam:

- You have 75 minutes to complete the exam.
- Formulae are provided on the last page of the exam packet. You may NOT use any other formula sheet.
- You might use any type of handheld calculator.
- Be sure to put a box around your final answers and clearly indicate your work to your grader.
- Partial credit can be given only if your work is clearly explained and labeled. No credit will be given if we can’t figure out which answer you are choosing, or which answer you want us to consider. If the answer marked does not obviously follow from the shown work, even if the answer is correct, you will not get credit for the answer.
- Multiple choice questions do not need to show any work to get credit.
- Have your TAMU ID ready when submitting your exam to the proctor.
- Please check that no pages are missing in your copy of the exam: pages are numbered and there should be 9 pages in total plus one page with formulae

Put your initials here after reading the above instructions: ______
Short Problems (25) _______
Problem 2 (25) _______
Problem 3 (25) _______
Problem 4 (25) _______
Total (100) _______
Problem 1 (25 points):
You do not need to show work to get credit. Generally, more than one correct answer is possible in these problems, but marking incorrect answers, in addition to the correct one(s), will reduce the credit you receive.

Problem 1.1. (5 pts) On a smooth horizontal floor, an object slides into a spring which is attached to another mass that is initially stationary. When the spring is most compressed, both objects are moving at the same speed. Ignoring friction, what is conserved in the interaction during which the spring is compressed?

- a) Momentum and mechanical energy
- b) Momentum only
- c) Momentum and potential energy
- d) Kinetic energy only
- e) Momentum and kinetic energy

Problem 1.2. (5 pts) Three uniform cylinders of the same length L but different radius 2R, R and 3R are made of the same type of material and placed next to each other as shown in the figure below. How far is the center of mass of the system from the center of the cylinder of radius R?

- a) 7/3 R
- b) 1/3 R
- c) 12/7 R
- d) 8/7 R
- e) 48/13 R
Problem 1.3. (5 pts) Moment of inertia of a uniform square plate for an axis going through and perpendicular to the center O (see figure) is $I = \frac{2}{3}MR^2$. A disk of material of radius $R$ is removed from the center. What is the new moment of inertia with the circular material removed?

a) $\pi/8 \text{ MR}^2$

b) $(1/3 - \pi/12) \text{ MR}^2$

c) $(\pi/4) \text{ MR}^2$

d) $(2/3 - \pi/8) \text{ MR}^2$

e) $(2/3 - \pi/12) \text{ MR}^2$

Problem 1.4. (5 pts) A turntable has a radius of 0.80 m and a moment of inertia of 2.0 $kg\cdot m^2$. The turntable is rotating with an angular velocity of 1.5 rad/s about a vertical axis through its center on frictionless bearings. A very small 0.40-kg ball is projected horizontally toward the turntable axis with a velocity of 3.0 m/s. The ball is caught by a cup-shaped mechanism on the rim of the turntable. The angular velocity of the turntable just after the ball is caught is closest to:

a) 0.75 rad/s

b) 0.30 rad/s

c) 0.94 rad/s

d) 2.1 rad/s

e) 1.3 rad/s
Problem 1.5. (5 pts) Jacques and George meet in the middle of a lake while paddling in their canoes. They come to a complete stop and talk for a while. When they are ready to leave, Jacques pushes George’s canoe with a force $\vec{F}$ to separate the two canoes. What is correct to say about the final momentum and kinetic energy of the system if we can neglect any resistance due to the water?

A. The final momentum is in the direction of $\vec{F}$ but the final kinetic energy is zero.

B. The final momentum is zero but the final kinetic energy is positive.

C. The final momentum is in the direction opposite of $\vec{F}$ but the final kinetic energy is zero.

D. The final momentum is zero and the final kinetic energy is zero.

E. The final momentum is in the direction of $\vec{F}$ and the final kinetic energy is positive.
Problem 2 (25 points):

A system shown on the right, consists of two pulleys, each attached at their centers to the wall with nails and able to rotate freely (without friction) around their axes, and a box of mass $m_B$. The pulley on the left is made of a uniform thin disk of radius $R$ and mass $M$ and, glued to it, a circular hoop of radius $r$ and mass $m$ (the centers of the cylinder and the disk coincide). The pulley on the right is a uniform disk of mass $m$ and radius $r$. The box hangs from a string wound over the pulley on the right, and then around the circular hoop, which is part of the pulley on the left, as shown in the figure. The system is initially at rest with the bottom of the box at height $h$ above the floor. The string is massless and does not stretch.

A. (7 pts) What is the moment of inertia $I_L$ of the pulley on the left?

B. (10 pts) The system is released from rest and the box drops to the floor. What is the velocity of the box just before it strikes the floor? Assume that during the motion the string does not slip over the surfaces of the pulleys. If you wish, you can express your answer in terms of $I_L$, which you had to evaluate in part A, and other given variables in the problem.

C. (8 pts) What is angular acceleration $\alpha_L$ of the pulley on the left while the system is in motion? If you wish, you can express your answer in terms of $I_L$, which you had to evaluate in part A, and other given variables in the problem.

Name__________________________________       Section Number ______________________
You can use this page if you need extra space to show your work
Problem 3 (25 points):

Two asteroids of equal mass in the asteroid belt between Mars and Jupiter collide with a glancing blow. Asteroid A, which was initially traveling at $v_o = 50.0 \text{ m/s}$ with respect to an inertial frame in which asteroid B was at rest, is deflected $\theta_A = 10.0^\circ$ from its original direction, while asteroid B travels at $\theta_B = 75.0^\circ$ to the original direction of asteroid A as shown in the figure.

A. (10 pts) Find the speed of asteroid A after the collision.

B. (7 pts) Find the speed of asteroid B after the collision.

C. (8 pts) What fraction of the original kinetic energy dissipates during this collision?
Problem 4 (25 points):

Jack Sparrow is forced to walk the plank by Davy Jones. The 5.00-m long uniform plank weighs 300 N and is supported from above by a cable, like a drawbridge, as shown in the figure to the right. The cable is 2.5 meters long and is attached to the side of the ship making an angle of 60° with the vertical axis.

A. (5 pts) Turn the figure into a free-body diagram showing all relevant forces for the case when Jack is at the very end of the plank.

B. (10 pts) If the support cable can withstand up to 3400 N of tension before breaking, what is the heaviest that Jack can weigh if it won’t break before he reaches the end of the plank?

C. (10 pts) Let’s say Jack actually weighs this much; what is the magnitude of the force that the hinge provides to keep the plank anchored to the ship when Jack is at the end of the plank?
Phys 218 — Exam III Formulae

**Vectors and Trigonometry:**

\[
\begin{align*}
\vec{h}_{\text{adj}} &= h \cos \theta = h \sin \phi & h^2 &= h_{\text{adj}}^2 + h_{\text{opp}}^2 \\
\vec{h}_{\text{opp}} &= h \sin \theta = h \cos \phi & \tan \theta &= \frac{h_{\text{opp}}}{h_{\text{adj}}} \\
\vec{A} &= A_x \hat{i} + A_y \hat{j} + A_z \hat{k} & \vec{A} &= \frac{\vec{A}}{|\vec{A}|} \\
\vec{A} \cdot \vec{B} &= A_x B_x + A_y B_y + A_z B_z = AB \cos \theta = A_B = AB \parallel \\
\vec{A} \times \vec{B} &= (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j} + (A_x B_y - A_y B_x) \hat{k} \\
|\vec{A} \times \vec{B}| &= AB \sin \theta = A_B = AB \perp
\end{align*}
\]

**Kinematics:**

\[
\begin{align*}
\text{translational} & & \text{rotational} \\
(\vec{v}) &= \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} & (\omega) &= \frac{\vec{a}_2 - \vec{a}_1}{t_2 - t_1} \\
(\vec{a}) &= \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} & (\alpha) &= \frac{\vec{a}_2 - \vec{a}_1}{t_2 - t_1} \\
\vec{r}(t) &= \vec{r}_0 + \int_0^t \vec{v}(t') \, dt' & \theta(t) &= \theta_0 + \int_0^t \omega(t') \, dt' \\
\vec{v}(t) &= \vec{v}_0 + \int_0^t \vec{a}(t') \, dt' & \vec{v}(t) &= \vec{v}_0 + \int_0^t \vec{a}(t') \, dt' \\
\vec{v}_2^2 &= \vec{v}_1^2 + 2a_x (x - x_0) & \omega_2^2 &= \omega_0^2 + 2\alpha(\theta - \theta_0) \\
& \quad \text{(and similarly for } y \text{ and } z) & & \omega(t) = \omega_0 + \alpha t \\
\vec{r}(t) &= \vec{r}_0 + \frac{1}{2}(\vec{v}_1 + \vec{v}_f)t & \theta(t) &= \theta_0 + \frac{1}{2}(\omega_1 + \omega_f)t \\
& \quad \text{constant (linear/angular) acceleration only} & & \theta(t) = \theta_0 + \frac{1}{2}\alpha t^2 \\
\end{align*}
\]

**Energy and Momentum:**

\[
\begin{align*}
\text{translational} & & \text{rotational} \\
\vec{p} &= \vec{r} \times \vec{F} \quad \text{and} \quad |\vec{p}| = F_L r = Fl \\
K &= \frac{1}{2} M v^2 & K_{\text{rot}} &= \frac{1}{2} I_{\text{tot}} \omega^2 \\
W &= \int \vec{F} \cdot d\vec{r} \quad \text{force} \quad \Delta \vec{r} & W &= \int \tau \, d\theta \quad \text{torque} \quad \tau \Delta \theta \\
P &= \frac{dW}{dt} = \vec{F} \cdot \vec{v} & P &= \frac{dW}{dt} = \vec{F} \cdot \vec{\omega} \\
\vec{p}_{\text{cm}} &= m_1 \vec{v}_1 + m_2 \vec{v}_2 + \ldots & \vec{L} &= \sum \vec{r} \times \vec{p} \\
& = M \vec{v}_{\text{cm}} & = I_1 \vec{\omega}_1 + I_2 \vec{\omega}_2 + \ldots \\
\vec{J} &= \int \vec{F} \, dt = \Delta \vec{p} & = I_{\text{tot}} \vec{\omega} \\
\sum \vec{F}_{\text{ext}} &= M \vec{a}_{\text{cm}} = \frac{d\vec{p}_{\text{cm}}}{dt} & \sum \vec{r}_{\text{ext}} = I_{\text{tot}} \vec{a} = \frac{d\vec{L}}{dt} \\
\sum \vec{F}_{\text{int}} &= 0 & \sum \vec{r}_{\text{int}} &= 0 \\
& \quad \text{if } \sum F_{\text{ext,x}} = 0, \quad p_{\text{cm},x} = \text{const} & \quad \text{if } \sum r_{\text{ext},x} = 0, \quad L_z = \text{const} \\
& \quad \text{Work-energy and potential energy} \quad \text{—} & \quad \text{—} \\
W &= \Delta K = E_{\text{tot,f}} + W_{\text{other}} \quad U = -\int \vec{F} \cdot d\vec{r} ; \quad U_{\text{grav}} = M g y_{\text{cm}} ; \quad U_{\text{elas}} = \frac{1}{2} k \Delta x^2 \\
F_{\text{x}}(x) &= -dU(x)/dx & \vec{F} &= -\nabla U = -\left(\frac{dU}{dx} \hat{i} + \frac{dU}{dy} \hat{j} + \frac{dU}{dz} \hat{k}\right) \\
\end{align*}
\]

**Quadratic:**

\[
ax^2 + bx + c = 0 \quad \Rightarrow \quad x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

**Derivatives:**

\[
\frac{d}{dt} (at^n) = nat^{n-1} \quad \frac{d}{dt} \sin at = a \cos at \quad \frac{d}{dt} \cos at = -a \sin at
\]

**Integrals:**

\[
\begin{align*}
\text{if } f(t) &= at^n, \text{ then } & \int f(t) \, dt &= \frac{a}{n+1} (t^{n+1} - t_0^{n+1}) \\
\int \sin at \, dt &= \frac{1}{a} \cos at & \int \cos at \, dt &= \frac{1}{a} \sin at
\end{align*}
\]

**Constants/Conversions:**

\[
\begin{align*}
g &= 9.80 \, \text{m/s}^2 = 32.15 \, \text{ft/s}^2 \quad \text{(on Earth’s surface)} \\
G &= 6.674 \times 10^{-11} \, \text{N m}^2 / \text{kg}^2 \\
1 \text{ mi} &= 1.609 \text{ km} \quad 1 \text{ lb} = 4.448 \text{ N} \\
1 \text{ rev} &= 360^\circ = 2\pi \text{ radians}
\end{align*}
\]

**Circular motion:**

\[
\begin{align*}
a_{\text{rad}} &= \frac{v^2}{r} \quad a_{\text{tan}} &= \frac{d|\vec{v}|}{dt} = R \alpha \\
T &= \frac{2\pi R}{v} & s &= R \theta & v_{\text{tan}} &= R \omega
\end{align*}
\]

**Relative velocity:**

\[
\vec{v}_{A/C} = \vec{v}_{A/B} + \vec{v}_{B/C} \quad \vec{v}_{A/B} = -\vec{v}_{B/A}
\]

**Forces:**

\[
\begin{align*}
\text{Newton’s:} \quad \sum \vec{F} &= m \vec{a}, \quad \vec{F}_{\text{on A}} = -\vec{F}_{\text{on B}} \quad \text{(on Earth’s surface)} \\
\text{Hooke’s:} \quad \vec{F} &= -k \Delta x \\
\text{friction:} \quad |\vec{f}_s| &\leq \mu_s |\vec{n}|, \quad |\vec{f}_k| = \mu_k |\vec{n}|
\end{align*}
\]

**Centre-of-mass:**

\[
\vec{r}_{\text{cm}} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \ldots + m_n \vec{r}_n}{m_1 + m_2 + \ldots + m_n}
\]

\[
\begin{align*}
& \quad \text{(and similarly for } \vec{v} \text{ and } \vec{a})
\end{align*}
\]

**Gravity:**

\[
\vec{F}_{\text{grav}} = -G \frac{M_1 M_2}{r^2} \hat{r} \quad U_{\text{grav}} = -G \frac{M_1 M_2}{r}
\]

**Kepler’s Laws:**

\[
\begin{align*}
1^\text{st}: \quad & \tan \text{ent} \text{ion} \\
& \text{Perihelion} \quad P \quad \text{Aphelion} \quad O \\
& \text{S} \text{S}’ \\
& \text{ea} \quad \text{ea} \\
a \quad a
\end{align*}
\]

\[
2^\text{nd}: \quad \vec{r} \times \vec{v} = \text{constant}
\]

\[
3^\text{rd}: \quad T = \frac{2\pi a^{3/2}}{\sqrt{GM}}
\]
Moments of inertia:

- Slender rod, axis through centre: $I = \frac{1}{12}ML^2$
- Slender rod, axis through one end: $I = \frac{1}{3}ML^2$
- Rectangular plate, axis through centre: $I = \frac{1}{12}Ma^2 + \frac{1}{12}Mb^2$
- Thin rectangular plate, axis along edge: $I = \frac{1}{3}Ma^2$
- Hollow cylinder: $I = \frac{1}{2}M(R_1^2 + R_2^2)$
- Solid cylinder: $I = \frac{1}{2}MR^2$
- Thin-walled hollow cylinder: $I = \frac{2}{3}MR^2$
- Solid sphere: $I = \frac{2}{5}MR^2$
- Thin-walled hollow sphere: $I = \frac{2}{3}MR^2$

→ For a point-like particle of mass $M$ a distance $R$ from the axis of rotation: $I = MR^2$

→ Parallel axis theorem: $I_p = I_{cm} + Md^2$