Homework 2

due: Wednesday September 20

• **Problem 1** [25 pts]
  Solve
  \[ \frac{d^2y}{dx^2} + 4y = x^2 \sin 2x. \]  

• **Problem 2** [25 pts]
  Verify that
  \[ \nabla^2 \psi(r, \theta, \phi) + \left[ k^2 + f(r) + \frac{1}{r^2} g(\theta) + \frac{1}{r^2 \sin^2 \theta} h(\phi) \right] \psi(r, \theta, \phi) = 0 \]  
is separable in spherical coordinates. Here \( k^2 \) is a constant.

• **Problem 3** [25 pts]
  Find the general solution of
  \[ A(x)y''(x) + A'(x)y'(x) + \frac{y(x)}{A(x)} = 0 \]  

  where \( A(x) \) is a known function and \( y(x) \) is unknown.

• **Problem 4** [25 pts]
  Given is the differential equation
  \[ (1 - x^3)y'' - 6x^2y' - 6xy = 0, \]  

  with the boundary conditions \( y = 1, y' = 0 \) at \( x = 0 \). Find the solution by assuming that \( y \) is a power series expanded around \( x = 0 \). Sum the series and verify that the sum satisfies the differential equation.