4. [25pts] A block of mass \( m \) slides down to the bottom from rest at the top of an incline of height \( h \) and an incline angle \( \theta = 37^\circ \). The coefficient of kinetic friction between the incline and the block is \( \mu = \frac{3}{5} \).

(It is useful to note that \( \sin(37^\circ) \approx \frac{3}{5} \) and \( \cos(37^\circ) \approx \frac{4}{5} \).)

(a) What is the work done by the gravity force? (4pts)

\[
W_g = mg \sin \theta \left( \frac{h}{\sin \theta} \right) = mgh \sqrt{1}
\]

(b) What is the work done by the normal force? (3pts)

Since the normal force is \( \perp \) to the direction of motion, no work is done by the normal force.

\[
W_{\text{normal}} = 0
\]

(c) What is the work done by the friction force? (4pts)

\[
W_f = -\mu (mg \cos \theta) \left( \frac{h}{\sin \theta} \right) = -\frac{2}{5} (mg) \left( \frac{4}{5} \right) h \left( \frac{5}{3} \right) = -\frac{8}{15} mgh
\]

(d) Use the work-energy principle to determine the speed of the block just before it hits the bottom. (4pts)

\[
\frac{1}{2}mv^2 - 0 = mgh - \frac{8}{15} mgh
\]

\[
\therefore \sqrt{\frac{1}{2}v^2} = \left( \frac{15 - 8}{15} \right) mg \quad \Rightarrow \quad v = \sqrt{\frac{1}{2}gh} \quad \Rightarrow \quad v = 3.02 \sqrt{h}
\]

(f) Let us consider a one-dimensional system. An object of mass \( m \) is pulled by a net force \( F \), moving from \( x = 0 \) to \( x = a \). The net force may not be a constant so that the total work done by this net force is \( W = \int_0^a F \, dx \). Prove, starting from Newton’s second law of motion \( F = ma \), that \( W \) is equal to the change of the kinetic energy of the object (10pts)

\[
F = m \frac{dv}{dt}
\]

\[
F \cdot dx = m \frac{dv}{dt} \cdot dx
\]

\[
F \cdot dx = m \frac{dv}{dt} \cdot dx \cdot dt
\]

\[
F \, dv = mv \cdot dv
\]

\[
W = \int_0^a F \, dx = \int_v^0 \left( \frac{1}{2}mv^2 \right) \, dv = \left[ \frac{1}{2}mv^2 \right]_v^i = \frac{1}{2}mv_i^2 - \frac{1}{2}mv_f^2 = \Delta K
\]