USEFUL INFORMATION

If \( f(x) = kx^n \) \( \frac{df}{dx} = nkx^{n-1} \)

If \( f(x) = kx^n \) \( \int_A^B f(x)dx = \frac{1}{n+1} k(B^{n+1} - A^{n+1}) \)

If \( f(x) = kx^n \) \( \int f(x)dx = \frac{1}{n+1} kx^{n+1} + C \)

\[ \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_{tot} \cdot d\vec{r} = \frac{1}{2} mv^2(\vec{r}_2) - \frac{1}{2} mv^2(\vec{r}_1) \]

If \( \vec{F} \) is conservative:

\[ \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = -[U(\vec{r}_2) - U(\vec{r}_1)] \]

and

\[ F_x = -\frac{\partial U}{\partial x} \quad F_y = -\frac{\partial U}{\partial y} \]

1.
2.
3.
4.
1. (25 points) A massless, unstretchable rope goes over a frictionless pulley. A man, mass $m_1$, is holding on at one side and the other end is attached to a block of mass $m_2$. The block is on top of a scale. The man begins to climb up the rope, at $t = 0$, with an acceleration that increases with time according to $a = c_1 t$ where $c_1$ is a known, positive constant. At what time would the block still be at rest but the scale read zero?

Free Body Diagrams (If appropriate). Law or Definition

Application

Result
2. (25 points) The motion of a small object of mass $m$ is observed in the region along the $x$ axis between $x = 0$ and $x = L$. The kinetic energy is measured and found to vary with its position according to $KE(x) = K_0(1 - \frac{x^2}{L^2})$ where $K_0$ is a known constant. Assuming that only a conservative force acts on the object, what is the force?

Free Body Diagram (If appropriate). Law or Definition.

Application

Result
3. (25 points) A small block of mass $m$ is at rest on a table. The coefficient of friction between the block and the table is $\mu$. (The coefficient of static friction is equal to the coefficient of kinetic friction in this problem.) A horizontal force is applied at $t = 0$ which has magnitude $\beta t$ where $\beta$ is a known constant. Find the block's velocity as a function of time.

\[ \beta t \rightarrow m \]

Free Body Diagrams (If appropriate). Law or Definition

Application

Result
4. (25 points) A small block, mass \( m \), is attached to a spring which is connected to the floor. The unstretched length of the spring is \( L \). The mass is released from rest at the point where the spring is unstretched. The spring is not a perfect Physics 218 spring but instead the magnitude of the force exerted by the spring is \( k_1 \) times the amount the spring is stretched or compressed plus \( k_2 \) times the cube of the amount stretched or compressed. Here \( k_1 \) and \( k_2 \) are known, positive constants. How far down will the block go before starting back up?

Free Body Diagram (If appropriate). Law or Definition

Application

Result (An equation for the distance is sufficient. Do not solve.)