1. (25 points) Three charges are placed as shown.

![Diagram of three charges](image)

The distances $a$ and $b$ are known. The charge at the origin is known and negative, $-q_1$. The charge $q_2$ at $x = 0, y = b$ is unknown. The charge $q_3$ at $x = a, y = 0$ is unknown. What must be the unknown charges $q_2$ and $q_3$ if the electric force on a positive charge $q_4$ at $x = a, y = b$ is to be zero?

**Law**

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{|q_1 q_2|}{r^2} \hat{r}$$

**Application**

$$\cos \Theta = \frac{a}{\sqrt{a^2 + b^2}}$$

$$F_x = \frac{1}{4\pi\varepsilon_0} \left( \frac{q_2 q_4}{a^2} - \frac{q_1 q_4}{(a^2 + b^2)^{3/2}} \right) = 0$$

$$F_y = \frac{1}{4\pi\varepsilon_0} \left( \frac{q_3 q_4}{b^2} - \frac{q_1 q_4}{(a^2 + b^2)^{3/2}} \right) = 0$$

\[
\frac{q_2}{a^2} - \frac{q_1 a}{(a^2 + b^2)^{3/2}} = 0 \Rightarrow q_2 = \frac{q_1 a^3}{(a^2 + b^2)^{3/2}}
\]

\[
\frac{q_3}{b^2} - \frac{q_1 b}{(a^2 + b^2)^{3/2}} = 0 \Rightarrow q_3 = \frac{q_1 b^3}{(a^2 + b^2)^{3/2}}
\]

**Result**
2. (25 points) An amount of charge \( Q \) is uniformly distributed along a semi-circle of radius \( R \) whose center is a distance \( A \) from the origin. What point charge would have to be placed at the origin so that the electric field at the center of the semi-circle would be zero?

![Diagram of a semi-circle with charge distribution and radius R, and a point charge Q at the origin.]

\[
\vec{E} = \vec{E}_1 + \vec{E}_2
\]

\[
\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \vec{r}
\]

**Application**

\[
\vec{E}_1 = \frac{1}{4\pi\varepsilon_0} \frac{q}{A^2} \vec{i}_x
\]

\[
d\vec{E}_2 = \frac{1}{4\pi\varepsilon_0} \frac{dQ}{R^2} \cos\theta \vec{r}
\]

\[
E_{2y} = 0 \quad \text{from symmetry}
\]

\[
dQ = \frac{Q}{\pi R} R \, d\theta = \frac{Q}{\pi} \, d\theta
\]

\[
E_{2x} = \frac{2}{4\pi\varepsilon_0} \int_0^{\pi/2} \frac{Q}{R^2} \cos\theta \, d\theta = \frac{Q}{2\pi^2\varepsilon_0 R^2} \sin\theta \bigg|_0^{\pi/2} = \frac{Q}{2\pi^2\varepsilon_0 R^2}
\]

\[
\frac{1}{4\pi\varepsilon_0} \frac{q}{A^2} + \frac{Q}{2\pi^2\varepsilon_0 R^2} = 0
\]

**Result**

\[
q = -\frac{2Q A^2}{\pi \varepsilon_0 R^2}
\]
3. (25 points) An amount of charge $Q$ is uniformly distributed along a semi-circle of radius $R$ whose center is at the origin. Find the electric potential function at the point $x = a$ assuming the value of the electric potential at infinity is zero.

\[ dV(r) = \frac{1}{\eta \epsilon_0} \frac{dQ}{r} \]

**Application**

\[ dQ = \frac{Q}{\pi R} r \, d\theta = \frac{Q}{\pi} \, d\theta \]

\[ r = \sqrt{(x + a)^2 + y^2}; \quad x = R \cos \theta \]
\[ y = R \sin \theta \]

\[ V = \frac{1}{\eta \epsilon_0} \frac{Q}{\pi} \frac{1}{\sqrt{(x + a)^2 + y^2}} \, d\theta = \]

\[ = \frac{1}{\eta \epsilon_0} \frac{Q}{\pi} \frac{1}{\sqrt{R^2 \cos^2 \theta + a^2 + R^2 \sin^2 \theta}} \, d\theta \]

**Result** What is the value of the electric potential you found above for the special case where $a = 0$?

\[ V = \frac{1}{\eta \epsilon_0} \frac{Q}{R} \, d\theta = \frac{1}{\eta \epsilon_0} \frac{Q}{R} \pi = \frac{Q}{\eta \epsilon_0 R} \]
4. (25 points) Suppose the force exerted on a point test charge \( q_0 \) by a point charge \( Q \) was given by

\[
\vec{F} = C \frac{q_0 Q}{r^6} \hat{r}
\]

where, just like in the Coulomb force, \( r \) is the distance between the points, \( \hat{r} \) is along the line from one point to the other and \( C \) is a positive, known constant. The force is repulsive for these two positive charges. Find the flux of \( \vec{E} \) corresponding to this force for a surface which is a sphere of radius \( R \) with center at the origin. Also find the difference in the electric potential difference between a point \( 2R \) from the origin and a point infinitely far from the origin.

\[
\Phi = \int \vec{E} \cdot d\vec{S} \quad ; \quad V(\vec{r}_2) - V(\vec{r}_1) = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{E} \cdot d\vec{r}
\]

**Law**

\[
\Phi = \int \vec{E} \cdot d\vec{S}
\]

**Application**

\[
\vec{F} = C \frac{q_0 Q}{r^6} \hat{r}
\]

\[
E_r = C \frac{Q}{r^6}
\]

\[
S = \pi R^2
\]

\[
\Phi = \frac{C Q}{R^6} \pi R^2 = \frac{C \pi Q}{R^4}
\]

\[
V(xR) - V(\infty) = -\int_0^{xR} C \frac{Q}{r^6} dr = \left. C \frac{Q}{5r^5} \right|_0^{xR} = \frac{2CQ}{5xR^5}
\]

**Result**

\[
\Phi = \frac{C Q}{5(2R)^5} = \frac{1}{160} \frac{C Q}{R^5}
\]

\[
\Phi = \frac{C Q}{R^4
\]