Monostatic lidar/radar invisibility using coated spheres

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Abstract: The Lorenz-Mie theory is revisited to explicitly include materials whose permeability is different from unity. The expansion coefficients of the scattered field are given for light scattering by both homogeneous and coated spheres. It is shown that the backscatter is exactly zero if the impedance of the spherical particles is equal to the intrinsic impedance of the surrounding medium. If spherical particles are sufficiently large, the zero backscatter can be explained as impedance matching using the asymptotic expression for the radar backscattering cross section. In the case of a coated sphere, the shell can be regarded as a cloak if the product of the thickness and the imaginary part of the refractive index of the outer shell is large.

OCIS codes: (280.1350) Atmospheric scattering; (290.5850) Scattering, particles.

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1. Introduction

Electromagnetic cloaking is an active research topic that has recently drawn a great deal of attention [1, 2, 3]. A coordinate transformation approach [1, 4] and a conformal mapping approach [2] have been employed to generate a cloak with complex material properties which makes the particle invisible when viewed externally. A two dimensional cylindrical cloak was demonstrated by Schurig et al. [3]. Schurig et al. simulated this invisible cloak in the geometric optics limit [5]. Cummer et al. [6] and Zolla et al. [7] used "COSMOL Multiphysics," a commercial software package based on a finite element-based electromagnetics solver to study a full-wave 2D cloaking system. Chen et al. [8] reported an analytical solution to the interaction of electromagnetic wave with a 3D metamaterial cloak. Zhang et al. [9] have demonstrated that the incident electromagnetic waves induced magnetic and electric surface currents at the inner boundary of a 2D cylindrical cloak. Other work related to invisible cloaks used plasmonic resonance to achieve transparency [10, 11].

In Ref. [8], the solution is based on expanding the electromagnetic waves in terms of vector spherical harmonics. The expansion coefficients are determined from the boundary conditions. This approach is essentially a generalization of the Lorenz-Mie theory [12]. The expansion coefficients of the scattered wave turn out to be zero if the following permittivity and permeability are used [1]:

$$\begin{aligned}\overline{\overline{\varepsilon}} &= \varepsilon_r(r)\hat{r}\hat{r} + \varepsilon_t(\hat{\theta}\hat{\theta} + \hat{\phi}\hat{\phi}),\\ \overline{\overline{\mu}} &= \mu_r(r)\hat{r}\hat{r} + \mu_t(\hat{\theta}\hat{\theta} + \hat{\phi}\hat{\phi}),\end{aligned} \tag{1}$$

where $\varepsilon_t/\varepsilon_0 = R_2/(R_2 - R_1)$, $\varepsilon_r = \varepsilon_t(r - R_1)^2/r^2$, $\mu_t/\mu_0 = \varepsilon_t/\varepsilon_0$, and $\mu_r/\mu_t = \varepsilon_r/\varepsilon_t$; R_1 and R_2 are the inner and outer radii of the two concentric spheres, respectively; ε_0 and μ_0 are the permittivity and permeability of the surrounding medium; $\hat{r}$, $\hat{\theta}$, and $\hat{\phi}$ are the unit vectors of the spherical coordinates. It is worthy noting that the medium defined by Eq. (1) is lossless. If loss exists, the cloak becomes visible and only the exact backscatter is zero. Chen et al. [8] articulated that "This unique property of the spherical cloak indicates the cloaked object can still completely hide from monostatic lidar/radar detection."

We studied the case carefully and found that zero backscatter is not a unique property of the spherical cloak represented by Eq. (1). In fact, one only needs to require $\eta_1 = \eta_0$, which is also called the impedance matching condition, for a homogeneous spherical particle to achieve zero backscatter, where

$$\eta_i = \sqrt{\frac{\mu_i}{\varepsilon_i}}, \quad i = 1, 0, \tag{2}$$

is the impedance of the spherical particle; subscripts 1 and 0 denote the quantities for the sphere and surrounding medium, respectively. For a coated sphere, the backscatter is also zero if the impedance matching condition $\eta_i = \eta_0$ is satisfied for each i^{th} layer of the coated sphere. Furthermore, one can speculate from physical intuition that the backscatter of a coated sphere is still approximately zero even if the impedance matching condition is violated for the inner core,

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given that its outer layer satisfies the condition and the imaginary part of its refractive index is sufficiently large. This is because the electromagnetic wave cannot penetrate into the inner region. Thus no matter what is inside, as long as it is a passive medium, the scattering pattern is essentially the same. This speculation is proved by numerical study and it can be used to make a cloak invisible for monostatic lidar/radar detection. The condition of this type of cloak is not as strict as that suggested in Ref. [1] if only backscatter is concerned. Materials with impedance matching the surrounding medium can be made by using the Maxwell-Garnett mixing rule [13] for compounds from known material properties [14].

2. Theory and formulas

The Lorenz-Mie theory is treated in detail in several classical texts [15, 16]. For a homogeneous sphere, we define:

$$x = ka, \qquad \mathrm{m} = \sqrt{\frac{\varepsilon_1 \mu_1}{\varepsilon_0 \mu_0}}, \tag{3}$$

where x is the size parameter of the sphere; k is the magnitude of the wave vector in the surrounding medium; a is the radius of the sphere; m is the relative refractive index of the sphere; ε is the permittivity; μ is the permeability. Consider the scattering of a x-polarized electromagnetic plane wave by this sphere. The propagation direction of the incident wave is along the z axis. The incident, scattered, and internal fields can be expanded in terms of the vector spherical harmonics. The expansion coefficients are determined by the boundary conditions. Eqs. (4.53) in Ref. [16] gave the general expressions of the expansion coefficients of the scattered wave. Bohren and Huffman [16] assumed $\mu_1/\mu_0 = 1$ and obtained Eqs. (4.88) in Ref. [16] for numerical calculations. To derive the corresponding equations of Eqs. (4.88) in Ref. [16] for the case of $\mu_1/\mu_0 \neq 1$, the relative impedance η is introduced:

$$\eta = \frac{\eta_1}{\eta_0} = \sqrt{\frac{\mu_1}{\varepsilon_1}} / \sqrt{\frac{\mu_0}{\varepsilon_0}}. \tag{4}$$

Following steps similar to those in Ref. [16], the expansion coefficients of scattered field can be written as:

$$\begin{aligned} a_n &= \frac{[D_n(\mathrm{m}x)\eta + n/x]\psi_n(x) - \psi_{n-1}(x)}{[D_n(\mathrm{m}x)\eta + n/x]\xi_n(x) - \xi_{n-1}(x)}, \\ b_n &= \frac{[D_n(\mathrm{m}x)/\eta + n/x]\psi_n(x) - \psi_{n-1}(x)}{[D_n(\mathrm{m}x)/\eta + n/x]\xi_n(x) - \xi_{n-1}(x)}, \end{aligned} \tag{5}$$

where the standard nomenclature for the special functions involved in the Lorenz-Mie theory is adopted [16]. Specifically, $\psi_n(\rho) = \rho j_n(\rho)$ and $\xi_n(\rho) = \rho h_n^{(1)}(\rho)$ are the Riccati-Bessel functions; j_n and $h_n^{(1)} = j_n + i y_n$ are the spherical Bessel functions of the first and third kinds, respectively; y_n is the spherical Bessel function of the second kind. $D_n(\rho) = d \ln \psi_n(\rho)/d\rho$ is the logarithmic derivative of the Riccati-Bessel function ψ_n.

An immediate observation of Eq. (5) is that $a_n = b_n$ if the relative impedance $\eta = 1$. From Eqs. (4.74) in Ref. [16], the complex amplitude scattering matrix elements can be written as:

$$S_1 = S_2 = \sum_n \frac{2n+1}{n(n+1)} a_n(\pi_n + \tau_n), \tag{6}$$

which are equal to zero at $\theta = 180^\circ$ since $\pi_n(180^\circ) + \tau_n(180^\circ) = 0$ for all n. The radar backscattering cross section σ_b is defined as the area which is 4π times the differential scattering cross section for scattering into a unit solid angle around the backscattering direction [16]. It

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is then evident that σ_b is equal to zero for $\eta = 1$. For large absorbing spherical particles, the asymptotic expression of the radar back scattering efficiency $Q_b = \sigma_b/\pi a^2$ has a value of (Eq. 4.83 in Ref. [16])

$$\lim_{x\to\infty} Q_b = R(0^\circ), \tag{7}$$

where $R(0^\circ)$ is the reflectance at normal incidence at the surface of the sphere. $R(0^\circ)$ vanishes if the impedance matching is satisfied, namely, $\eta = 1$ (see pp. 306, Ref. [17]).

For spherical particles, the scattering matrix can be written in terms of the amplitude scattering matrix as follows (see Eqs. (4.77) in Ref. [16]):

$$S_{11} = \frac{|S_2|^2 + |S_1|^2}{2}, \qquad S_{12} = \frac{|S_2|^2 - |S_1|^2}{2},$$
$$S_{33} = \frac{S_2^* S_1 + S_2 S_1^*}{2}, \qquad S_{34} = \frac{i(S_1 S_2^* - S_2 S_1^*)}{2}. \tag{8}$$

Eqs. (8) shows $S_{12} = S_{34} = 0$ and $S_{11} = S_{33}$ for particles with $\eta = 1$. Only diagonal elements survive for this case.

Light scattering by a coated sphere is also treated in Ref. [16]. Eqs. (8.1) in Ref. [16] give the relations for the eight expansion coefficients of the fields. The permeability $\mu = 1$ is then assumed for the particle and surrounding medium to obtain a_n and b_n. To reinstate the dependence of μ in a_n and b_n, the following definitions are convenient:

$$x = ka, \quad \mathrm{m}_1 = \sqrt{\frac{\varepsilon_1\mu_1}{\varepsilon_3\mu_3}}, \quad \eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}\Big/\sqrt{\frac{\mu_3}{\varepsilon_3}},$$
$$y = kb, \quad \mathrm{m}_2 = \sqrt{\frac{\varepsilon_2\mu_2}{\varepsilon_3\mu_3}}, \quad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}\Big/\sqrt{\frac{\mu_3}{\varepsilon_3}}, \tag{9}$$

where a and b are the inner and outer radii of the coated sphere, respectively; $\mathrm{m}_i, i = 1,2$ are the relative refractive indices in the region i; the subscripts $i = 1,2$ denote that the quantities are for the regions of $0 < r < a$ and $a < r < b$, respectively; ε_3 and μ_3 are the permittivity and permeability of the surrounding medium, respectively. The expansion coefficients of the scattered field can be solved from Eqs. (8.1) in Ref. [16]; given by:

$$a_n = \frac{[\eta_2\tilde{D}_n + n/y]\psi_n(y) - \psi_{n-1}(y)}{[\eta_2\tilde{D}_n + n/y]\xi_n(y) - \xi_{n-1}(y)},$$
$$b_n = \frac{[\tilde{G}_n/\eta_2 + n/y]\psi_n(y) - \psi_{n-1}(y)}{[\tilde{G}_n/\eta_2 + n/y]\xi_n(y) - \xi_{n-1}(y)}, \tag{10}$$

where

$$\tilde{D}_n = \frac{D_n(\mathrm{m}_2 y) - A_n\chi_n'(\mathrm{m}_2 y)/\psi_n(\mathrm{m}_2 y)}{1 - A_n\chi_n(\mathrm{m}_2 y)/\psi_n(\mathrm{m}_2 y)},$$
$$\tilde{G}_n = \frac{D_n(\mathrm{m}_2 y) - B_n\chi_n'(\mathrm{m}_2 y)/\psi_n(\mathrm{m}_2 y)}{1 - B_n\chi_n(\mathrm{m}_2 y)/\psi_n(\mathrm{m}_2 y)},$$
$$A_n = \psi_n(\mathrm{m}_2 x)\frac{\eta_1 D_n(\mathrm{m}_1 x) - \eta_2 D_n(\mathrm{m}_2 x)}{\eta_1 D_n(\mathrm{m}_1 x)\chi_n(\mathrm{m}_2 x) - \eta_2\chi_n'(\mathrm{m}_2 x)},$$
$$B_n = \psi_n(\mathrm{m}_2 x)\frac{\eta_1 D_n(\mathrm{m}_2 x) - \eta_2 D_n(\mathrm{m}_1 x)}{\eta_1\chi_n'(\mathrm{m}_2 x) - \eta_2 D_n(\mathrm{m}_1 x)\chi_n(\mathrm{m}_2 x)}. \tag{11}$$

$\chi_n(\rho) = -\rho y_n(\rho)$ is used in Eqs. (11). If $\eta_1 = \eta_2 = 1$, it is obvious that $A_n = B_n$, $\tilde{D}_n = \tilde{G}_n$, and $a_n = b_n$. According to Eqs. (8), it is the same as the homogeneous sphere since $S_{11}(\theta = 180^\circ) = 0$. Furthermore, $S_{12} = S_{34} = 0$ and $S_{11} = S_{33}$ also hold for coated spheres if $\eta_1 = \eta_2$.

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3. Numerical results

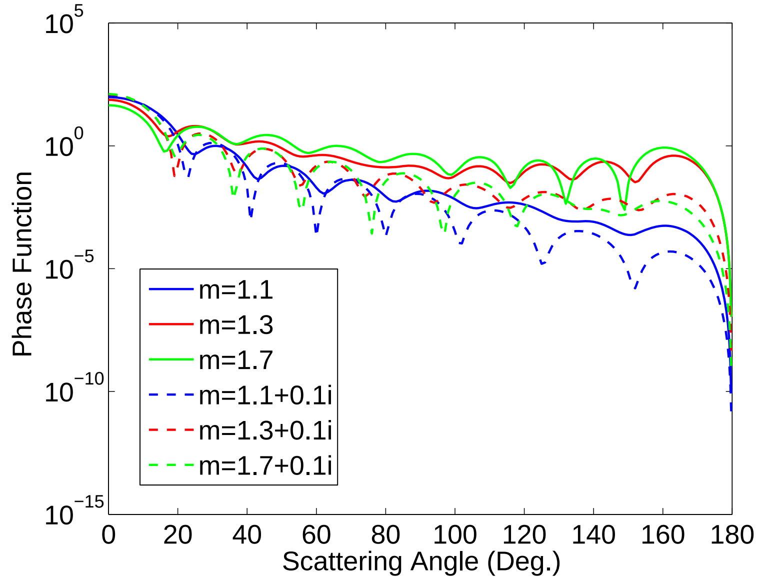

Fig. 1. The phase function as a function of the scattering angle for homogeneous spherical particles. $\eta = 1$ and the size parameter $x = 10$ are used. The refractive indices plotted are $\mathrm{m} = 1.1, 1.3, 1.7, 1.1+0.1\mathrm{i}, 1.3+0.1\mathrm{i}, 1.7+0.1\mathrm{i}$

Table 1. Extinction and scattering efficiencies for the cases in Fig. 1

	m=1.1	m=1.3	m=1.7	m=1.1 + 0.1i	m=1.3 + 0.1i	m=1.7 + 0.1i
Q_{ext}	1.7129	2.6575	1.7716	1.9358	2.4718	2.3655
Q_{sca}				0.8926	1.2506	1.1172

Two codes were written on the basis of the theory and formulas in Sec. 2 to calculate the scattering properties of homogeneous and coated spheres whose relative permeabilities are not unity. The various numerical techniques suggested by Bohren and Huffman [16], Wiscombe [18, 19], and Toon and Ackerman [20] are employed in this study. For homogeneous spheres with $\eta = 1$, it would be interesting to show how fast S_{11} approaches zero around $\theta = 180^\circ$ for different values of m. Figure 1 plots the phase function, the normalized scattering matrix element S_{11}, for $\mathrm{m} = 1.1, 1.3, 1.7, 1.1+0.1\mathrm{i}, 1.3+0.1\mathrm{i}$, and $1.7+0.1\mathrm{i}$. In this computation, $\eta = 1$ and the size parameter $x = 10$ are assumed. Evidently, the phase function approaches zero faster in the region of $170^\circ < \theta < 180^\circ$ as the relative refractive index decreases. For $\mathrm{m} = 1.3$ and 1.7, the rates of decrease of the phase functions are almost the same. The rate of decrease of the phase functions stops if the refractive index reaches a certain value. In addition, an imaginary part of the refractive index causes the phase function to decrease faster. Table 1 shows the extinction efficiencies Q_{ext} and scattering efficiencies Q_{sca} for the cases in Fig. 1. The cases with zero imaginary part of the refractive indices, the extinction and scattering efficiencies are equal. It is interesting that the extinction efficiency for $\mathrm{m} = 1.3+0.1\mathrm{i}$ is smaller than that of $\mathrm{m} = 1.3$, while the extinction efficiencies of $\mathrm{m} = 1.1+0.1\mathrm{i}$ and $\mathrm{m} = 1.7+0.1\mathrm{i}$ are larger than those of $\mathrm{m} = 1.1$ and $\mathrm{m} = 1.7$, respectively.

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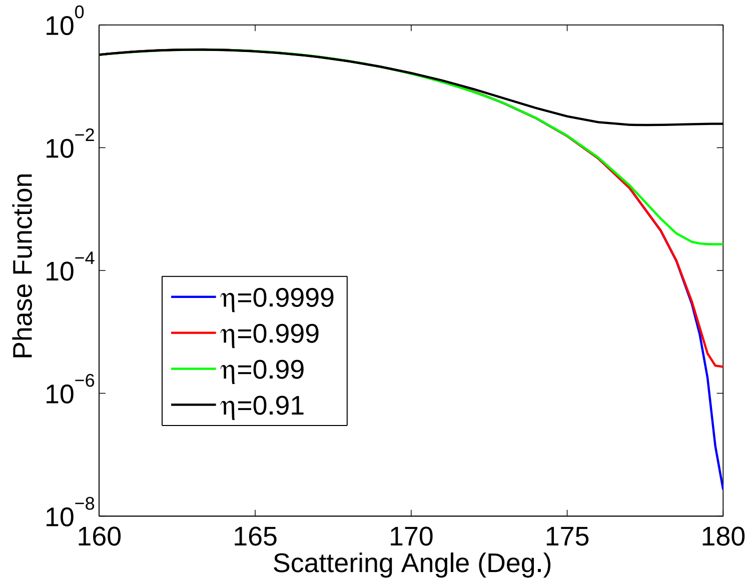

Fig. 2. The phase function as a function of the scattering angle for homogeneous spherical particles. The size parameter is $x = 10$. The refractive indices is m = 1.3. $\eta = 0.9999, 0.999, 0.99, 0.91$ are plotted.

Table 2. Extinction/scattering efficiencies for cases in Fig. 2

η=0.9999	η=0.999	η=0.99	η=0.91
2.6575	2.6575	2.6575	2.6594

To understand how the sensitivity of the phase function depends on η, Fig. 2 shows the phase functions for spherical particles with $\eta = 0.9999, 0.999, 0.99, 0.91$ for scattering angles ranging from 160° to 180°. A size parameter of $x = 10$ and a relative refractive index of $\mathrm{m} = 1.3$ are used. It is shown that the backscatter is small if η remains close to 1. As η deviates from 1, the backscatter also gradually deviates from 0. Table 2 shows the extinction/scattering efficiencies for cases in Fig. 2. The extinction and scattering efficiencies are equal for the four cases because the imaginary part of the refractive indices are zero. The extinction efficiencies do not have significant changes until η reaches 0.91.

For coated spheres, the phase function is also zero in the backscattering direction if $\eta_{1,2} = 1$ is satisfied for each layer, which means that it can be completely hidden from monostatic lidar/radar detection. An important question is to find under what conditions a coated sphere can still hide from monostatic lidar/radar detection even if its core region has $\eta_1 \neq 1$. If these conditions are found, the shell of a coated sphere can be regarded as a cloak. From physical intuition, it is speculated that these conditions may include a thick outer shell with a large imaginary refractive index. Ref. [21] has shown the distribution of the internal electric field of a homogeneous sphere for different values of refractive index. It was shown that the electric field is mainly distributed around the boundary of the particle for a large refractive index of $\mathrm{m} = 7.1499 + 2.914i$. The physical reason is that the large absorption caused by the large imaginary part prevents the penetration of electromagnetic waves into the particle. It can be concluded

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that the electromagnetic waves cannot "see" the inner region of a particle provided it has a large imaginary refractive index. Applying this rule to a coated sphere, we have the following conditions under which a coated sphere can still hide from a monostatic lidar/radar even if the inner region has $\eta_1 \neq 1$:

$$\eta_2 = 1, \quad \tau = k(b-a)\mathrm{Im}(\mathrm{m}_2) = (y-x)\mathrm{Im}(\mathrm{m}_2) >> 1, \tag{12}$$

where k is the wave vector in the surrounding medium; $x = ka$ and $y = kb$ are the size parameters of the inner and outer spheres, respectively; $\mathrm{Im}(\mathrm{m}_2)$ is the imaginary part of the relative refractive index of the outer layer.

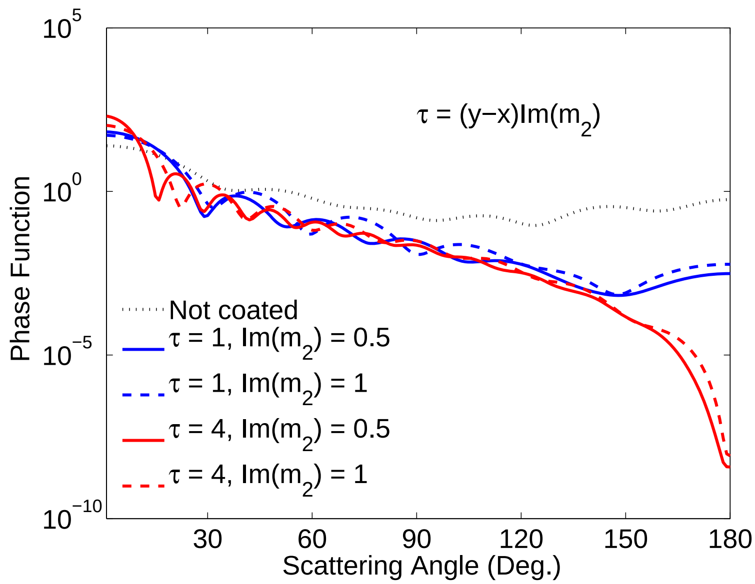

Fig. 3. The phase function as a function of the scattering angle for coated spherical particles. The size of the inner sphere is $x = 5.$ and $\mathrm{m}_1 = 1.5$, $\eta_1 = 2/3$. Two refractive indices for the outer sphere are chosen: $\mathrm{m}_2 = 2 + 0.5\mathrm{i}$ and $\mathrm{m}_2 = 2 + \mathrm{i}$. $\eta_2 = 1$. For each of the refractive indices, a proper value of y is used to produce $\tau = 1$ and $\tau = 4$ according to $\tau = (y-x)\mathrm{Im}(\mathrm{m}_2)$.

Table 3. Extinction and scattering efficiencies for cases in Fig. 3. Also shown are the ratios of the extinction and scattering cross sections of the coated particles and those of the "Not coated" case.

	Not coated	$\mathrm{Im}(\mathrm{m}_2) = 0.5$		$\mathrm{Im}(\mathrm{m}_2) = 1$	
		$\tau = 1$	$\tau = 4$	$\tau = 1$	$\tau = 4$
Q_{ext}	3.9278	2.5314	2.3368	2.3572	2.4121
Q_{sca}		1.1852	1.1507	0.9987	1.1544
C_{ext}/C_{ext}^{Nc}	–	1.2632	4.0218	0.8642	1.9897
C_{sca}/C_{sca}^{Nc}	–	0.5914	1.9804	0.3661	0.9522

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Figure 3 shows the phase functions for four coated spheres. The refractive indices for the outer shell are taken as $m_2 = 2+0.5i$ and $m_2 = 2+i$. $\eta_2 = 1$ is used to insure the outer layer produces a zero backscatter. The size parameters for the inner sphere is $x = 5$. A refractive index of $m_1 = 1.5$ is assumed for the inner core to test its sensitivity. The impedance for the inner sphere is $\eta_1 = 2/3$. Thus the permeability of the inner core is taken to be 1. We noticed that $\tau = (y-x)\mathrm{Im}(m_2)$ is the critical factor to achieve a zero backscatter. In Fig. 3, two values of $\tau = 1$ and 4 are chosen to test the sensitivity. For each value of τ and $\mathrm{Im}(m_2)$, the corresponding value of y can be determined by $\tau = (y-x)\mathrm{Im}(m_2)$. The phase function for the inner sphere without the coated sphere is also shown for comparison. It is evident from Fig. 3 that the coated sphere decreases the phase function at the backscatter. The phase functions for $\tau = 4$ have negligible values while the phase functions for $\tau = 1$ still have significant values at the backscatter. This feature confirms that τ is the dominant factor that determines the magnitude of the backscatter. This is a remarkable result because $\tau = k(b-a)\mathrm{Im}(m_2) = 4$ is just a fairly moderate value and the backscatter has already been reduced by several orders of magnitude. Table 3 shows the extinction and scattering efficiencies for cases in Fig. 3. The particles shown in Fig. 3 all have different sizes, which makes it hard to interpret the efficiencies. To overcome this problem, the ratio of the extinction/scattering cross sections for the coated particle and the "Not coated" particle is introduced.

$$\frac{C_{ext,sca}}{C^{Nc}_{ext,sca}} = \frac{Q_{ext,sca} \cdot y^2}{Q^{Nc}_{ext,sca} \cdot x^2}, \tag{13}$$

where $C^{Nc}_{ext,sca}$ is the extinction/scattering cross section for the "Not coated" case; $C_{ext,sca}$ is the extinction/scattering cross section for the coated particle. The proportionality between the size parameter and the radius of a spherical particle is used to get the right hand side of Eq. (13). The size parameter x of the "Not coated" case is equal to that of the cores of the coated particles. Equation (13) represents the ratio of the power eliminated/scattered from the incident light for the coated and "Not coated" particles. Table 3 shows the extinction cross sections for the coated particles generally larger than that of the "Not coated" particle. The only exception is in the case of $\tau = 1$ and $\mathrm{Im}(m_2) = 0.5$. It means that the negligible backscatters are achieved at the cost of larger extinction cross sections.

4. Conclusions

The expansion coefficients of the scattered field are given for light scattering by both homogeneous and coated spheres whose relative permeability values are not assumed to be 1. Two Fortran codes were written to calculate the scattering matrix for these particles. An interesting feature is that a zero backscatter for homogeneous and coated spheres can be achieved if the impedance for each region of the particle is equal to that of the surrounding medium. If the inner core of a coated sphere does not satisfy this condition, the back scattering can still be approximately zero under the conditions that the impedance $\eta_2 = 1$ holds for its outer layer and $\tau = (y-x)\mathrm{Im}(m_2)$ is sufficiently large. In other words, this coated sphere can act as an invisible cloak for monostatic lidar/radar detection. Furthermore, the extinction cross section for the coated sphere is generally larger than the "Not coated" particle.

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