Problem 1. (15 points) A glass tank containing methanol \((n = 1.329)\) has walls 3.00 cm thick. The glass has an index of refraction of 1.550. Light from the outside strikes the glass at an angle of 41.30° with the normal to the glass as shown.

(a) Find the angle the light makes with the normal in the methanol.

(b) The tank is now emptied and refilled with an unknown liquid. If the light incident at the same angle as in part (a) and enters the liquid in the tank at an angle of 20.0°, what is the index of refraction of this unknown liquid?

\[
\sin \theta_m = \frac{1.00 \sin 41.30°}{1.329} = 0.4966
\]

\[
\theta_m = 29.99°
\]

\[
\sin \theta_m = \frac{1.00 \sin 41.30°}{1.93}
\]

\[
\theta_m = 29.99°
\]

Problem 2. (20 points) A candle is placed at the center of curvature of a concave mirror with a focal length of 10.0 cm as shown in the figure. 85.0 cm to the right of the candle is located a thin lens with a focal length of 35.0 cm. The candle is viewed from the right through the lens. The lens forms two images of the candle. The first image is formed by the light passing directly through the lens, and the second is formed from the light that is reflected off the mirror and then through the lens.

(a) For each of these two images, draw a principal-ray diagram that locates the image.

(b) For each image answer the following:
   a. Where is the image located?
   b. Is the image real or virtual?
   c. Is the image inverted or erect?
Problem 3. (15 points) Two thin parallel slits that are 0.0129 mm apart, are illuminated by a laser beam of wavelength 600 nm.

(a) On a very large distant screen, what is the total number of bright fringes, including the central fringe and those on both sides of it that will be seen? (Hint: what is the largest value that \( \sin \theta \) can have?)

(b) At what angle, relative to the original direction of the beam, will the fringe that is most distant from the central bright fringe be located?

\[
\sin \theta_{\text{max}} = \frac{2 \lambda}{d} = 9.75
\]

\[
\theta_{\text{max}} = 77.3^\circ
\]

Problem 4. (15 points) A plate of glass 11.00 cm long is placed in contact with a second plate and is held at a small angle with it by a metal strip 0.0800 mm thick placed under one end. The space between the glass plates is filled with air. The glass is illuminated from above with a light of wavelength 656 nm. How many interference fringes are observed per centimeter in the reflected light?

Path difference = 2t (air wedge)

Total phase change for two reflected waves:

\[
\Delta \phi = \frac{2t}{\lambda} + \frac{\pi}{2} = \frac{4t \pi}{\lambda} (\text{bright})
\]

\[
\Delta \phi = \frac{2t}{\lambda} \pi (\text{dark})
\]

Dark fringe located

\[
\frac{2t}{\lambda} \pi = n \pi
\]

Total number of dark fringes

\[
\frac{2t}{\lambda} = \frac{2}{6.5 \times 10^{-7}} = 243.9 (\text{a} \text{ cm})
\]

Interference fringes/cm = \( \frac{243.9}{11} = 22.17 \)
Problem 5. (15 points) Two headlights of an approaching car are 140.0 cm apart. At what maximum distance will an eye be able to resolve them? Assume that the pupil diameter is 5.00 mm and that the light has a wavelength of 550 nm. Assume also that diffraction effects limit this resolution and that the Rayleigh Condition defines the ability to resolve the two lights as being distinct sources.

\[
\sin \theta = 1.22 \frac{\lambda}{D}
\]

\[
\sin \theta = 1.22 \left( \frac{550 \times 10^{-9} \text{m}}{5 \times 10^{-3} \text{m}} \right)
\]

\[
\theta = 0.00134 \text{ radians}
\]

\[
\text{(Angular separation)} = 1.34 \times 10^{-4} \text{ radians}
\]

Problem 6. (20 points) Your boss asks you to design a diffraction grating that will disperse the first order visible spectrum through an angular range of 12.0°.

(a) What must the number of slits per centimeter be for this grating?

(b) At what angles will the first-order visible spectrum begin and end? Use the bounds for the spectrum 400.0 nm to 700.0 nm.

Note, you might find this relationship useful:

\[
\sin(\alpha + \beta) = (\sin \alpha)(\cos \beta) + (\cos \alpha)(\sin \beta)
\]

\[
d \sin \theta = n \lambda \text{ for bright spots, } n \neq 0 \text{ for the two colors}
\]

\[
d \sin \theta_v = \lambda_v \text{ and } d \sin \theta_r = \lambda_r = 700 \text{ nm}
\]

We also know that \( \theta_r - \theta_v = 12^\circ \)

So \( d \sin \theta_v = \lambda_v = d \sin(\theta_v + 12^\circ) \)

2 equations

with 2 unknowns

\[
\lambda_v = d \sin \theta_v
\]

Substitute \( d = \frac{\lambda_v}{\sin \theta_v} \) into first equation to get

\[
\lambda_v = \left( \frac{\lambda_v}{\sin \theta_v} \right) \left( \sin \theta_v \cos 12^\circ + \cos \theta_v \sin 12^\circ \right)
\]

\[
\lambda_v = \cos 12^\circ + \cos \theta_v \sin 12^\circ
\]

\[
\cos \theta_v = 3.712 \text{rad} \quad \theta_v = 15.07^\circ
\]

\[
\lambda_v = 700 \text{ nm}
\]

\[
d = \frac{\lambda_v}{\sin \theta_v} = 1.54 \times 10^{-3} \text{m}^{-1}
\]

\[
\text{Air wavelength} = 6562 \text{ nm}
\]