Diffraction

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Chapter 36
Diffraction
and coherent sources

- Reminder of the form of E&M Waves

\[ E(x,t) = E_0 \sin(kx - \omega t + \phi), \text{ with} \]
\[ k = \frac{2\pi}{\lambda} \quad \text{and} \quad \omega = 2\pi f \]
Finding the resultant E field

\[ E_p \] is the resultant field

(c) As (b), but in the limit that the slit is subdivided into infinitely many strips
Intensity pattern for a single slit

\[ I = I_0 \left\{ \frac{\sin[\pi a(\sin \theta) / \lambda]}{\pi a(\sin \theta) / \lambda} \right\}^2 \]

Intensity in single slit diffraction

\[ \frac{a \sin \theta}{\lambda} = m \]

for the position of the dark fringes
\[ \theta_1 = \frac{\lambda}{a} \]

for the position of 1st dark fringe
Diffraction patterns for 2 slits

\[ I = I_0 \cos^2 \left( \frac{\phi}{2} \right) \left[ \frac{\sin(\beta/2)}{\beta/2} \right]^2 \]

for two slits of finite width

\[ \phi = \frac{2\pi d}{\lambda} \sin \theta \quad \text{and} \quad \beta = \frac{2\pi a}{\lambda} \sin \theta \]
Now let’s look at what happens with several slits.

\[ d \sin \theta = m \lambda \]

for constructive interference

Slits are narrower than \( \lambda \) so the diffraction patterns are broad and modulated by the interference effects from multiple slits.
Diffraction patterns for different numbers of slits

(a) $N = 2$

(b) $N = 8$

(c) $N = 16$
Diffraction gratings

\[ d \sin \theta = m \lambda \]

for constructive interference

Bright bands will be at angles satisfying this condition.
Grating Spectrograph

Can change the orientation of the beam to the grating to map out the spectrum of the sample beam of light.

1. Light from telescope is sent along fiber-optic cables (not shown) and emerges here.

2. Light strikes concave mirror and emerges as a beam of parallel rays.

3. Light passes through diffraction grating.

4. Lenses direct diffracted light onto a second concave mirror.

5. Concave mirror reflects light to a focus.

6. An electronic detector (like the one in a digital camera) records the spectrum.
X-Ray Diffraction

Crystals are a three dimensional diffraction grating.
The “Bragg Condition” $2d\sin\theta = m\lambda$ gives constructive interference
Circular apertures and resolving power

\[ \sin \theta_1 = 1.22 \frac{\lambda}{d} \]

angle of the first dark ring from a circular aperture.
Circular apertures and resolving power an example

Problem 36.48: A converging lens 7.20 cm in diameter has a focal length of 300 mm. If resolution is diffraction limited, how far away can an object be if points on it 4.00 mm apart are to be resolved.

Use $\lambda = 550$ nm.

$$\sin \theta_1 = 1.22 \frac{\lambda}{d} = 1.22 \left( \frac{550 E - 9 m}{7.20 E - 2 m} \right) = 9.32E - 6$$

angle between the two points at the diffraction limit.

$$\frac{4.00E - 3}{\text{distance to object}} = 9.32E - 6; \quad d = 429 \text{ m}$$