Review of SHM, Mechanical Waves and Sound

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Chapters
13, 15 and 16
Periodic motion is motion that repeats itself in a definite cycle. It occurs whenever a body has a stable equilibrium position and a restoring force that acts when it is displaced from equilibrium. Period $T$ is the time for one cycle. Frequency $f$ is the number of cycles per unit time. Angular frequency $\omega$ is $2\pi$ times the frequency. (See Example 13.1)

\[ f = \frac{1}{T} \quad T = \frac{1}{f} \quad (13.1) \]
\[ \omega = 2\pi f = \frac{2\pi}{T} \quad (13.2) \]

The circle of reference construction uses a rotating vector called a phasor, having a length equal to the amplitude of the motion. Its projection on the horizontal axis represents the actual motion of a body in simple harmonic motion.

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If the net force is a restoring force $F_x$ that is directly proportional to the displacement $x$, the motion is called simple harmonic motion (SHM). In many cases this condition is satisfied if the displacement from equilibrium is small.

$$F_x = -kx \quad (13.3)$$

$$a_x = \frac{F_x}{m} = -\frac{k}{m}x \quad (13.4)$$

The angular frequency, frequency, and period in SHM do not depend on the amplitude, but only on the mass $m$ and force constant $k$. (See Examples 13.2, 13.6, and 13.7)

$$\omega = \sqrt{\frac{k}{m}} \quad (13.10)$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad (13.11)$$

$$T = \frac{1}{f} = 2\pi \sqrt{\frac{m}{k}} \quad (13.12)$$

In SHM, the displacement, velocity, and acceleration are sinusoidal functions of time. The angular frequency is $\omega = \sqrt{k/m}$; the amplitude $A$ and phase angle $\phi$ are determined by the initial position and velocity of the body. (See Example 13.3)

$$x = A \cos (\omega t + \phi) \quad (13.13)$$
Energy is conserved in SHM. The total energy can be expressed in terms of the force constant $k$ and amplitude $A$.

(See Examples 13.4 and 13.5)

\[ E = \frac{1}{2} m u_x^2 + \frac{1}{2} k x^2 = \frac{1}{2} k A^2 = \text{constant} \quad (13.21) \]
Other forms of SHM

In angular simple harmonic motion, the frequency and angular frequency are related to the moment of inertia $I$ and the torsion constant $\kappa$.

\[ \omega = \sqrt{\frac{\kappa}{I}} \quad \text{and} \quad f = \frac{1}{2\pi} \sqrt{\frac{\kappa}{I}} \quad (13.24) \]

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A simple pendulum consists of a point mass $m$ at the end of a massless string of length $L$. Its motion is approximately simple harmonic for sufficiently small amplitude; the angular frequency, frequency, and period then depend only on $g$ and $L$, not on the mass or amplitude.

(See Examples 13.8)

\[ \omega = \sqrt{\frac{g}{L}} \quad (13.32) \]
\[ f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{g}{L}} \quad (13.33) \]
\[ T = \frac{2\pi}{\omega} = \frac{1}{f} = 2\pi \sqrt{\frac{L}{g}} \quad (13.34) \]

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Damped Harmonic Motion and Resonance

When a damping force \( F_x = -bv_x \) proportional to velocity is added to a simple harmonic oscillator, the motion is called a damped oscillation. For a relatively small damping force, the motion is sinusoidal with a decaying amplitude and an angular frequency \( \omega' \) that is lower than it would be without damping. This situation occurs when \( b < 2\sqrt{km} \), a condition called underdamping. When \( b = 2\sqrt{km} \), the system is critically damped and no longer oscillates. When \( b > 2\sqrt{km} \), the system is overdamped.

\[
x = Ae^{-(\frac{b}{2m})t} \cos \omega' t \quad (13.42)
\]

\[
\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \quad (13.43)
\]

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When a sinusoidally varying driving force is added to a damped harmonic oscillator, the resulting motion is called a forced oscillation. The amplitude is a function of driving frequency \( \omega_d \) and reaches a peak at a driving frequency close to the natural frequency of the system. This behavior is called resonance.

\[
A = \frac{F_{\text{max}}}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}} \quad (13.46)
\]

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Mechanical Waves (Chapter 15)

In a periodic wave, the motion of each point of the medium is periodic. A sinusoidal wave is a special periodic wave in which each point moves in simple harmonic motion. For any periodic wave, the frequency \( f \) is the number of cycles per unit time, the period \( T \) is the time for one cycle, the wavelength \( \lambda \) is the distance over which the wave pattern repeats, and the amplitude \( A \) is the maximum displacement of a particle in the medium. The product of \( \lambda \) and \( f \) equals the wave speed. (See Example 15.1)

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The speed of transverse waves on a string depends on the tension \( F \) and mass per unit length \( \mu \). (See Examples 15.3 and 15.4)

\[
v = \sqrt{\frac{F}{\mu}} \quad \text{(waves on a string)} \quad (15.13)
\]

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The wave function obeys a partial differential equation called the wave equation. 

\[
\frac{\partial^2 y(x, t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y(x, t)}{\partial t^2} \quad (15.12)
\]

The wave function \( y(x, t) \) describes the displacements of individual particles in the medium. Equations (15.3), (15.4), and (15.7) give the wave equation for a sinusoidal wave traveling in the +x-direction. If the wave is moving in the −x-direction, the minus signs in the cosine functions are replaced by plus signs. (See Example 15.2)

\[
y(x, t) = A \cos \left[ \omega \left( \frac{x}{v} - t \right) \right] = A \cos 2\pi f \left( \frac{x}{v} - t \right) \quad (15.3)
\]

\[
y(x, t) = A \cos 2\pi \left( \frac{x}{\lambda} - \frac{t}{T} \right) \quad (15.4)
\]

\[
y(x, t) = A \cos (kx - \omega t) \quad (15.7)
\]

where \( k = \frac{2\pi}{\lambda} \) and \( \omega = 2\pi f = \nu k \)

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Wave motion conveys energy from one region to another. For a sinusoidal mechanical wave, the average power $P_{av}$ is proportional to the square of the wave amplitude and the square of the frequency. For waves that spread out in three dimensions, the wave intensity $I$ is inversely proportional to the distance from the source. (See Examples 15.5 and 15.6)

$$P_{av} = \frac{1}{2} \sqrt{\mu F o^2 A^2}$$  \hspace{1cm} (15.25)

(average power, sinusoidal wave)

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$  \hspace{1cm} (15.26)

(inverse-square law for intensity)

Wave power versus time $t$ at coordinate $x = 0$
The Principle of Wave Superposition

A wave that reaches a boundary of the medium in which it propagates is reflected. The principle of superposition states that the total wave displacement at any point where two or more waves overlap is the sum of the displacements of the individual waves.

\[ y(x, t) = y_1(x, t) + y_2(x, t) \]

(principle of superposition) (15.27)

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Standing Waves on a String

When a sinusoidal wave is reflected from a fixed or free end of a stretched string, the incident and reflected waves combine to form a standing sinusoidal wave with nodes and antinodes. Adjacent nodes are spaced a distance $\lambda/2$ apart, as are adjacent antinodes. (See Example 15.7)

$$y(x, t) = (A_{sw} \sin kx) \sin \omega t$$

(standing wave on a string, fixed end at $x = 0$)  \hspace{1cm} (15.28)

When both ends of a string with length $L$ are held fixed, standing waves can occur only when $L$ is an integer multiple of $\lambda/2$. Each frequency and its associated vibration pattern is called a normal mode. The lowest frequency $f_1$ is called the fundamental frequency.

(See Examples 15.8 and 15.9)

$$f_n = n \frac{v}{2L} = nf_1 \quad (n = 1, 2, 3, \ldots)$$  \hspace{1cm} (15.33)

$$f_1 = \frac{1}{2L} \sqrt{\frac{F}{\mu}}$$  \hspace{1cm} (15.35)

(string fixed at both ends)

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Sound and Hearing
(Chapter 16)

Sound consists of longitudinal waves in a medium. A sinusoidal sound wave is characterized by its frequency \( f \) and wavelength \( \lambda \) (or angular frequency \( \omega \) and wave number \( k \)) and by its displacement amplitude \( A \). The pressure amplitude \( p_{\text{max}} \) is directly proportional to the displacement amplitude; the proportionality constant is the product of the wave number and the bulk modulus \( B \) of the wave medium. (See Examples 16.1 and 16.2)

\[
p_{\text{max}} = BkA \quad (16.5)
\]

(sinusoidal sound wave)

The speed of a longitudinal (sound) wave in a fluid depends on the bulk modulus \( B \) and density \( \rho \). If the fluid is an ideal gas, the speed can be expressed in terms of the temperature \( T \), molar mass \( M \), and ratio of heat capacities \( \gamma \) of the gas. The speed of longitudinal waves in a solid rod depends on the density and Young’s modulus \( Y \). (See Examples 16.3 through 16.5)

\[
v = \sqrt{\frac{B}{\rho}} \quad \text{(longitudinal wave in a fluid)} \quad (16.7)
\]

\[
v = \sqrt{\frac{\gamma RT}{M}} \quad \text{(sound wave in an ideal gas)} \quad (16.7)
\]

\[
v = \sqrt{\frac{Y}{\rho}} \quad \text{(longitudinal wave in a solid rod)} \quad (16.8)
\]

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Sound Intensity

The intensity $I$ of a sound wave is the time average rate at which energy is transported by the wave, per unit area. For a sinusoidal wave, the intensity can be expressed in terms of the displacement amplitude $A$ or the pressure amplitude $p_{\text{max}}$. (See Examples 16.6 through 16.9)

\[ I = \frac{1}{2} \sqrt{\rho B \omega^2 A^2} = \frac{p_{\text{max}}^2}{2\rho v} \]

\[ -\frac{p_{\text{max}}^2}{2\sqrt{\rho B}} \] (16.12), (16.14)

(intensity of a sinusoidal sound wave)

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The sound intensity level $\beta$ of a sound wave is a logarithmic measure of its intensity. It is measured relative to $I_0$, an arbitrary intensity defined to be $10^{-12}$ W/m$^2$. Sound intensity levels are expressed in decibels (dB). (See Examples 16.10 and 16.11)

\[ \beta = (10 \text{ dB}) \log \frac{I}{I_0} \]

(definition of sound intensity level) (16.15)

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Standing sound waves can be set up in a pipe or tube. A closed end is a displacement node and a pressure antinode; an open end is a displacement antinode and a pressure node. For a pipe of length $L$ open at both ends, the normal-mode frequencies are integer multiples of the sound speed divided by $2L$. For a stopped pipe (one that is open at only one end), the normal-mode frequencies are the odd multiples of the sound speed divided by $4L$. (See Examples 16.12 and 16.13)

$$f_n = \frac{nv}{2L} \quad (n = 1, 2, 3, \ldots) \quad (16.18)$$

(open pipe)

$$f_n = \frac{nv}{4L} \quad (n = 1, 3, 5, \ldots) \quad (16.22)$$

(stopped pipe)
Interference Effects with Sound

When two or more waves overlap in the same region of space, the resulting effects are called interference. The resulting amplitude can be either larger or smaller than the amplitude of each individual wave, depending on whether the waves are in phase (constructive interference) or out of phase (destructive interference). (See Example 16.15)

Beats are heard when two tones with slightly different frequencies $f_a$ and $f_b$ are sounded together. The beat frequency $f_{beat}$ is the difference between $f_a$ and $f_b$.

$$f_{beat} = f_a - f_b \quad (16.24)$$ (beat frequency)
The Doppler effect for sound is the frequency shift that occurs when there is motion of a source of sound, a listener, or both, relative to the medium. The source and listener frequencies $f_0$ and $f_L$ are related by the source and listener velocities $v_s$ and $v_L$ relative to the medium and to the speed of sound $v$.

(See Examples 16.16 through 16.20)

$\frac{f_L}{f_0} = \frac{v + v_L}{v + v_s}$  \hspace{1cm} (16.29)

(Doppler effect, moving source and moving listener)

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