Conversion from displacement to pressure in the wave

Undisturbed cylinder of air

\[ y_1 = y(x, t) \]

Disturbed cylinder of air

\[ y_2 = y(x + \Delta x, t) \]
Calculating the change in volume of the medium

\[
\Delta V = S(y_2 - y_1)
\]

\[
= S(y(x + \Delta x, t) - y(x, t))
\]

\[
\frac{\Delta V}{V} = \frac{(y(x + \Delta x, t) - y(x, t))}{\Delta x}
\]

\[
\frac{\Delta V}{V} = \lim_{\Delta x \to 0} \frac{(y(x + \Delta x, t) - y(x, t))}{\Delta x} = \frac{\partial y(x, t)}{\partial x}
\]
Relationship between volume change and pressure in a liquid

\[ p(x, t) = -B \frac{\partial y(x, t)}{\partial x} \]

where \( B \) is the bulk modulus of the material.

Note \( B \) is defined as: \( B = -\frac{p(x, t)}{(dV / V)} \).
Solving for the pressure in the disturbance gives...

So..

\[ p(x,t) = B k A \sin(kx - \omega t) \]

when \( y(x,t) = A \cos(kx - \omega t) \)

and \( p_{\text{max}} = B k A \)
Displacement vs Pressure

(a) Displacement $y$ versus position $x$ at $t = 0$

(b) Undisplaced particles
Displaced particles at $t = 0$

(c) Pressure fluctuation $p$ versus position $x$ at $t = 0$

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More on Bulk Modulus

- B for air at atmospheric pressure is $1.45 \times 10^5$ Pa
- Remember $1.0 \text{ Pa} = 1.0 \text{ N/m}^2$
- $1.0 \text{ Atmosphere pressure} = 1.013 \times 10^5 \text{ Pa}$
Speed of sound waves in a fluid…

\[ v = \sqrt{\frac{B}{\rho}} \]

For waves in water, \( B = (1/45.8) \times 10^{11} \) and density \( 1 \times 10^3 \) kg/m\(^3\). What is \( v \)?
Speed of sound waves in a solid bar (not bulk solids)...

\[ v = \sqrt{\frac{Y}{\rho}} \]
Speed of sound waves in an ideal gas...

\[ v = \sqrt{\frac{\gamma RT}{M}} \]

\[ \gamma = 1.40; \quad R = 8.314 \text{ J/mol} \cdot \text{K}; \]

and \( M \) is the molar mass
Standing sound waves

Certain sound frequencies produce a standing wave in tube:
\[ N = \text{displacement nodes} \]
\[ A = \text{displacement antinodes} \]
Standing waves in an organ pipe (open at both ends)

\[ f = \frac{nv}{2L} \]

for an open pipe with \( n \) an integer

(a) \( f_1 = \frac{v}{2L} \)

Open end always a displacement antinode

(b) \( f_2 = 2 \frac{v}{2L} = 2f_1 \)

(c) \( f_3 = 3 \frac{v}{2L} = 3f_1 \)
Standing waves for an organ pipe (closed at one end)

\[ f = \frac{nv}{4L} \]

for an stopped pipe and \( n \) an odd integer
Interference using sound

Two speakers emit waves in phase

Amplifier

\[ d_1, d_2, d_2 + \frac{\lambda}{2} \]

Point P
- Same path length from each speaker
- Waves arrive at P in phase

Point Q
- Difference in path lengths = \( \frac{\lambda}{2} \)
- Waves arrive at Q 1/2 cycle out of phase
Beats
The Doppler Effect
Beats