Sound and Hearing

March 6, 2012

Chapter 16
But first a final demonstration on standing waves

Remember from last lecture

\[ y_{\text{total}}(x,t) = y_1(x,t) + y_2(x,t) \]

with

\[ y_{\text{total}} \] also a solution to the wave equation
Standing Waves on a string

\[ y_1(x,t) = -A \cos(kx + \omega t) \text{ and} \]
\[ y_2(x,t) = A \cos(kx - \omega t) \]

then

\[ y_{total} = A(-\cos(kx + \omega t) + \cos(kx - \omega t)) \]

using \[ \cos(a \pm b) = \cos a \cos b \mp \sin a \sin b \]
Standing waves (continued)

\[ y_{\text{total}} = A(-\cos(kx + \omega t) + \cos(kx - \omega t)) \]
\[ y_{\text{total}} = (2A \sin(kx)) \sin(\omega t) \]
Location of the nodes for a standing wave on a string.

\[ y_{\text{total}} = (2A \sin kx) \sin \omega t \]

for a string fixed at \( x = 0 \),
the nodes will be located at points where
\[ kx = m\pi, \quad \text{with} \quad x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, 2\lambda, \ldots \]
Normal Modes of a String

The string is rigidly held at both ends, so we have two sets of boundary conditions to be satisfied.

\[ L = n \frac{\lambda}{2} \]
with \( f\lambda = \nu \) or \( \lambda = \frac{\nu}{f} \) and

where \( \nu = \sqrt{\frac{T}{\mu}} \) so by changing the tension in the rope, I change the velocity of propagation and as a result the wave length of the standing wave
Now on to Sound and Hearing

• We will be switching gears a bit to study longitudinal mechanical waves, like sound.

• As we have already noted, these types of waves also are governed by the wave equation, so there will be lots of similarities.

• However, since hearing is connected to these pressure waves, we will study some of the aspects peculiar to sound and hearing in more detail in this chapter.
Requirements and properties of sound

- Must have a medium. (air, water, gas, something to travel through)
- Audible range of sound for humans (20-20000 Hz).
- Pressure wave propagates in all directions.
- A typical disturbance has the form $y(x,t) = A \cos(kx-\omega t)$
A typical disturbance

All particles in fluid oscillate in SH with same amplitude and period

Two particles one wavelength apart oscillate in phase with each other

Wave travels one wavelength $\lambda$ in one period $T$
Conversion from displacement to pressure in the wave

Undisturbed cylinder of air

$y_1 = y(x, t)$

Disturbed cylinder of air

$y_2 = y(x + \Delta x, t)$

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Calculating the change in volume of the medium

\[
\Delta V = S(y_2 - y_1)
\]

\[
= S(y(x + \Delta x, t) - y(x, t))
\]

\[
\frac{\Delta V}{V} = \frac{S(y(x + \Delta x, t) - y(x, t))}{\Delta x}
\]

\[
\frac{\Delta V}{V} = \lim_{\Delta x \to 0} \frac{S(y(x + \Delta x, t) - y(x, t))}{\Delta x} = \frac{\partial y(x, t)}{\partial x}
\]
Relationship between volume change and pressure in a liquid

\[ p(x, t) = -B \frac{\partial y(x, t)}{\partial x} \]

where \( B \) is the bulk modulus of the material.

Note \( B \) is defined as: \( B = -\frac{p(x, t)}{dV/V} \)
Solving for the pressure in the disturbance gives...

So..

\[ p(x,t) = BkA \sin(kx - \omega t) \]

when \( y(x,t) = A \cos(kx - \omega t) \)

and \( p_{\text{max}} = BkA \)
Displacement vs Pressure

(a) Displacement $y$ versus position $x$ at $t = 0$

(b) Undisplaced particles

(c) Pressure fluctuation $p$ versus position $x$ at $t = 0$
More on Bulk Modulus

- B for air at atmospheric pressure is $1.45 \times 10^5$ Pa
- Remember $1.0$ Pa $= 1.0$ N/m$^2$
- $1.0$ Atmosphere pressure $= 1.013 \times 10^5$ Pa