Chapter 14

Periodic Motion

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Goals for Chapter 14

• To describe oscillations in terms of amplitude, period, frequency and angular frequency

• To do calculations with simple harmonic motion

• To analyze simple harmonic motion using energy

• To apply the ideas of simple harmonic motion to different physical situations

• To analyze the motion of a simple pendulum

• To examine the characteristics of a physical pendulum

• To explore how oscillations die out

• To learn how a driving force can cause resonance
Introduction

• Why do dogs walk faster than humans? Does it have anything to do with the characteristics of their legs?

• Many kinds of motion (such as a pendulum, musical vibrations, and pistons in car engines) repeat themselves. We call such behavior *periodic motion* or *oscillation*. 
What causes periodic motion?

- If a body attached to a spring is displaced from its equilibrium position, the spring exerts a \textit{restoring force} on it, which tends to restore the object to the equilibrium position. This force causes \textit{oscillation} of the system, or \textit{periodic motion}.

- Figure 14.2 at the right illustrates the restoring force $F_x$. 
Characteristics of periodic motion

- The *amplitude*, \( A \), is the maximum magnitude of displacement from equilibrium.

- The *period*, \( T \), is the time for one cycle.

- The *frequency*, \( f \), is the number of cycles per unit time.

- The *angular frequency*, \( \omega \), is \( 2\pi \) times the frequency: \( \omega = 2\pi f \).

- The frequency and period are reciprocals of each other: \( f = 1/T \) and \( T = 1/f \).

- Follow Example 14.1.
An object on the end of a spring is oscillating in simple harmonic motion. If the amplitude of oscillation is doubled, how does this affect the oscillation period $T$ and the object’s maximum speed $v_{\text{max}}$?

A. $T$ and $v_{\text{max}}$ both double.

B. $T$ remains the same and $v_{\text{max}}$ doubles.

C. $T$ and $v_{\text{max}}$ both remain the same.

D. $T$ doubles and $v_{\text{max}}$ remains the same.

E. $T$ remains the same and $v_{\text{max}}$ increases by a factor of $\sqrt{2}$. 
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Q14.2

This is an $x$-$t$ graph for an object in simple harmonic motion.

At which of the following times does the object have the most negative velocity $v_x$?

A. $t = T/4$
B. $t = T/2$
C. $t = 3T/4$
D. $t = T$
This is an $x$-$t$ graph for an object in simple harmonic motion.

At which of the following times does the object have the most negative velocity $v_x$?

A. $t = T/4$
B. $t = T/2$
C. $t = 3T/4$
D. $t = T$
This is an $x$-$t$ graph for an object connected to a spring and moving in simple harmonic motion.

At which of the following times is the potential energy of the spring the greatest?

A. $t = T/8$
B. $t = T/4$
C. $t = 3T/8$
D. $t = T/2$
E. more than one of the above
This is an $x$-$t$ graph for an object connected to a spring and moving in simple harmonic motion.

At which of the following times is the potential energy of the spring the greatest?

A. $t = T/8$

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E. more than one of the above
Simple harmonic motion (SHM)

- When the restoring force is *directly proportional* to the displacement from equilibrium, the resulting motion is called *simple harmonic motion* (SHM).

- An ideal spring obeys Hooke’s law, so the restoring force is $F_x = -kx$, which results in simple harmonic motion.

**Ideal case:** The restoring force obeys Hooke’s law ($F_x = -kx$), so the graph of $F_x$ versus $x$ is a straight line.

**Typical real case:** The restoring force deviates from Hooke’s law ...

... but $F_x = -kx$ can be a good approximation to the force if the displacement $x$ is sufficiently small.
SHM differential equation: $F = ma$

\[ F = -kx, \quad F = ma_x = m \frac{d^2x}{dt^2} \]

\[ -kx = m \frac{d^2x}{dt^2} \]

\[ x(t) = A \sin(\omega t) + B \cos(\omega t) = C \sin(\omega t + \phi) \]
Simple harmonic motion viewed as a projection

- Simple harmonic motion is the projection of uniform circular motion onto a diameter, as illustrated in Figure 14.5 below.

![Diagram of simple harmonic motion](image)

(a) Apparatus for creating the reference circle

- While the ball $Q$ on the turntable moves in uniform circular motion, its shadow $P$ moves back and forth on the screen in simple harmonic motion.

(b) An abstract representation of the motion in (a)

$\text{Ball moves in uniform circular motion.}$

$\text{Shadow moves back and forth on x-axis in SHM.}$

\[ x = A \cos \theta \]
Characteristics of SHM

- For a body vibrating by an ideal spring:

\[ \omega = \sqrt{\frac{k}{m}} \quad f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}} \]

- Follow Example 14.2 and Figure 14.8 below.
Displacement as a function of time in SHM

- The displacement as a function of time for SHM with phase angle $\phi$ is $x = A\cos(\omega t + \phi)$. (See Figure 14.9 at the right.)

- Changing $m$, $A$, or $k$ changes the graph of $x$ versus $t$, as shown below.

(a) Increasing $m$; same $A$ and $k$
Mass $m$ increases from curve 1 to 2 to 3. Increasing $m$ alone increases the period.

(b) Increasing $k$; same $A$ and $m$
Force constant $k$ increases from curve 1 to 2 to 3. Increasing $k$ alone decreases the period.

(c) Increasing $A$; same $k$ and $m$
Amplitude $A$ increases from curve 1 to 2 to 3. Changing $A$ alone has no effect on the period.
Graphs of displacement, velocity, and acceleration

- The graph below shows the effect of different phase angles.

- The graphs below show $x$, $v_x$, and $a_x$ for $\phi = \pi/3$.

(a) Displacement $x$ as a function of time $t$

(b) Velocity $v_x$ as a function of time $t$

(c) Acceleration $a_x$ as a function of time $t$
Behavior of $v_x$ and $a_x$ during one cycle

- Figure 14.13 at the right shows how $v_x$ and $a_x$ vary during one cycle.
- Refer to Problem-Solving Strategy 14.1.
- Follow Example 14.3.
Energy in SHM

- The total mechanical energy \( E = K + U \) is conserved in SHM:
  \[
  E = \frac{1}{2} m v_x^2 + \frac{1}{2} kx^2 = \frac{1}{2} kA^2 = \text{constant}
  \]
Energy diagrams for SHM

- Figure 14.15 below shows energy diagrams for SHM.
- Refer to Problem-Solving Strategy 14.2.
- Follow Example 14.4.

(a) The potential energy $U$ and total mechanical energy $E$ for a body in SHM as a function of displacement $x$

The total mechanical energy $E$ is constant.

(b) The same graph as in (a), showing kinetic energy $K$ as well

At $x = \pm A$ the energy is all potential; the kinetic energy is zero.

At $x = 0$ the energy is all kinetic; the potential energy is zero.

At these points the energy is half kinetic and half potential.
Energy and momentum in SHM

• Follow Example 14.5 using Figure 14.16.

(a)

(b)

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**Vertical SHM**

- If a body oscillates vertically from a spring, the restoring force has magnitude $kx$. Therefore the vertical motion is SHM.

- Follow Example 14.6.
Angular SHM

- A coil spring (see Figure 14.19 below) exerts a restoring torque $\tau_z = -\kappa \theta$, where $\kappa$ is called the torsion constant of the spring.

- The result is angular simple harmonic motion.
Vibrations of molecules

- Figure 14.20 shows two atoms having centers a distance \( r \) apart, with the equilibrium point at \( r = R_0 \).

- If they are displaced a small distance \( x \) from equilibrium, the restoring force is \( F_r = -(2U_0/R_0^2)x \), so \( k = 2U_0/R_0^2 \) and the motion is SHM.

- Follow Example 14.7.
The simple pendulum

- **A simple pendulum** consists of a point mass (the bob) suspended by a massless, unstretchable string.
- If the pendulum swings with a small amplitude $\theta$ with the vertical, its motion is simple harmonic. (See Figure 14.21 at the right.)
- Follow Example 14.8.
The physical pendulum

- A physical pendulum is any real pendulum that uses an extended body instead of a point-mass bob.
- For small amplitudes, its motion is simple harmonic. (See Figure 14.23 at the right.)
- Follow Example 14.9.
Tyrannosaurus rex and the physical pendulum

• We can model the leg of *Tyrannosaurus rex* as a physical pendulum.

• Follow Example 14.10 using Figure 14.24 below.
Damped oscillations

- Real-world systems have some dissipative forces that decrease the amplitude.
- The decrease in amplitude is called *damping* and the motion is called *damped oscillation*.
- Figure 14.26 at the right illustrates an oscillator with a small amount of damping.
- The mechanical energy of a damped oscillator decreases continuously.

With stronger damping (larger $b$):
- The amplitude (shown by the dashed curves) decreases more rapidly.
- The period $T$ increases
($T_0 =$ period with zero damping)
Forced oscillations and resonance

- A *forced oscillation* occurs if a *driving force* acts on an oscillator.
- *Resonance* occurs if the frequency of the driving force is near the *natural frequency* of the system. (See Figure 14.28 below.)