Short Problems (Circle the correct option) [NO Partial Credit] [20 Points]

A) (5 points) Three projectiles are launched with the same initial speed but different launch angles, from the ground (y=0), as shown in the figure. Rank the projectiles in the order in which they hit the ground (y=0).

i) A < B < C
ii) A = B = C
iii) C < B < A
iv) A < C < B
v) (A=C) > B
vi) (A=C) < B
vii) None of the above

B) (5 points) A particle is traveling in a circular orbit of radius $r$ as shown in the figure. At the point marked $P$, its speed is found to be decreasing with time.

From this information alone, what is the best we can say about the approximate direction and magnitude of the total acceleration of this particle?

Select the right direction (options i-viii) AND Select (circle) the right $> = \text{ or } <$ condition

i) Direction A and has a magnitude that is $> = \frac{v_P}{r}$
ii) In sector B and has a magnitude that is $> = \frac{v_P}{r}$
iii) Direction C and has a magnitude that is $> = \frac{v_P}{r}$
iv) In sector D and has a magnitude that is $> = \frac{v_P}{r}$
v) Direction E and has a magnitude that is $> = \frac{v_P}{r}$
vi) In sector F and has a magnitude that is $> = \frac{v_P}{r}$
vii) Direction G and has a magnitude that is $> = \frac{v_P}{r}$
viii) In sector H and has a magnitude that is $> = \frac{v_P}{r}$
C) **(5 points)** A space station being planned as a base of operations for interplanetary exploration consists of a large circular tube that is rotating about its center, like a large bicycle wheel, at constant speed. The outer rim of this tube has a diameter of \( D = 1.7 \text{ km} \). Calculate the linear (or tangential) speed that a point on the "outer rim" of the station must have, so as to enable the inhabitants of the station to feel the effect equal to the gravity at the earth’s surface (1.0 g)

\[
\frac{v^2}{R} = g \\
v = \sqrt{gR} \\
v = 91.3 \text{ m/s}
\]

**Options**

A. 3.3 m/s  
B. 4.1 m/s  
C. 9.8 m/s  
D. 23.5 m/s  
E. 29.2 m/s  
F. 73.5 m/s  
G. 91.3 m/s  
H. The correct answer is not listed in the above options A-G.

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D) **(5 points)** The position of a particle is changing with time as \( x(t) = 1.4 \ t^4 + 21.3 \ t^2 \). Its average acceleration between the times of \( t = 0.3 \text{ s} \) and \( t = 1.8 \text{ s} \) is

\[
a_{\text{ave}} = \frac{v(1.8) - v(0.3)}{1.8 - 0.3} \\
v(t) = 1.4(4)t^3 + 21.3(2)t \\
\therefore a_{\text{ave}} = \left[ \frac{1.4(4)(1.8)^3 + 21.3(2)(1.8)}{1.4(4)(0.3)^3 + 21.3(2)(0.3)} \right] \\
= \left[ \frac{32.16 + 76.48}{1.5} \right] \\
= \left[ \frac{108.64}{1.5} \right] \\
= 72.4 \text{ m/s}^2
\]

**Options**

A. 70.6 m/s²  
B. 64.3 m/s²  
C. 48.2 m/s²  
D. 42.6 m/s²  
E. 35.3 m/s²  
F. 21.3 m/s²  
G. The correct answer is not listed in the above options A-F.
Problem 2 (20 points)

A motorcycle starts from rest at \( x_0 = 0 \) and accelerates in the +x-direction with a constant acceleration \( a \). At the same time a truck driving in the same lane is at position \( x = +D \) traveling toward the motorcycle (in the −x-direction) with a constant speed \( v_0 \). Answer the following, in terms of the relevant combination of the given quantities, \( x_0, a, D \) and \( v_0 \) ONLY (note that not all of \( x_0, a, D \) and \( v_0 \) may be necessary for individual sub – parts).

A) Write the equation for the position of the motorcycle as a function of time and the equation for the position of the truck as a function of time.

B) At what time, \( t \), will the two vehicles collide?

C) For this part of the problem, consider that \( v_0 \) is not known. Give an expression for the magnitude of \( v_0 \) in order for the truck and motorcycle to collide at \( D/2 \).

\[
\begin{align*}
\text{A) } & \ x_m = \frac{1}{2} at^2 \\
& \ x_T = D - v_0 t \\
\text{B) when } x_m &= x_T \\
& \text{solve for } t \Rightarrow 2at = D - v_0 t \\
& \frac{1}{2} at^2 + v_0 t - D = 0 \\
& \text{solve quadratic eqn.} \\
& t = \frac{-v_0 \pm \sqrt{v_0^2 + 4 \left( \frac{1}{2} a \right) D}}{2 \left( \frac{1}{2} a \right)} \\
& t = \frac{-v_0 + \frac{1}{2} \sqrt{v_0^2 + 2aD}}{a} \\
& \text{(positive time)}
\end{align*}
\]

\[
\begin{align*}
\text{C) } & \ x_m = x_T = D/2 \\
& \frac{D}{2} = \frac{1}{2} \cdot \frac{1}{2} at^2 = x_m \\
& t = \sqrt[2]{\frac{D}{a}} \\
& \text{sub. into } \frac{D}{2} = D - v_0 t \Rightarrow x_T \\
& \text{solve for } v_0 \Rightarrow \frac{D}{2} - D = -v_0 \sqrt[2]{\frac{D}{a}} \\
& v_0 = \frac{D}{2} \sqrt[2]{\frac{a}{D}}
\end{align*}
\]
**Problem 3 (20 points)**

Gliding at a constant horizontal velocity of \( v_p = 21.0 \text{ m/s} \) at an altitude (height) of \( H = 170.12 \text{ m} \) from the surface of the earth, you notice your favorite professor a certain distance away on the ground looking toward you. You use your radar to find out how far your professor is and decide to give them a yummy moment by launching an M&M candy at a velocity of \( v_m = 5.0 \text{ m/s} \) at an angle \( \theta = 23^\circ \) with respect to the glider, so as to have the M&M enter your professor’s wide open awe-struck mouth which is at a height of 1.52 m from the surface of the earth.

Neglecting effects of wind and/or air resistance –

Calculate the magnitude of the displacement of the projectile from the point it is launched to your professor’s mouth as seen from the ground by your professor.

**Given:** \( v_p = 21.0 \frac{\text{m}}{\text{s}} \), \( v_m = 5.0 \frac{\text{m}}{\text{s}} \), \( \theta = 23^\circ \), \( H = 170.12 \text{ m} \), \( h_p = 1.52 \text{ m} \), \( |a_y| = 9.81 \frac{\text{m}}{\text{s}^2} \), \( |a_x| = 0 \text{ m/s}^2 \)

**To find:** Displacement \( |\Delta \vec{r}| = \sqrt{\Delta x^2 + \Delta y^2} \)

**CHOOSE** a coordinate-system with \(+x\) pointing in the direction of the glider’s velocity and \(-y\) downward. The \(y\) – displacement (not distance) \( \Delta y \) is easy to see from the problem statement.

It is \( \Delta y = -(H - h_p) = -168.6 \text{ m} \) *(Equation A)*

Since the accelerations in both \(x\) and \(y\) are constant in time, we can describe the \(x\) and \(y\) displacements for the M&M as the projectile, through --

\[
\Delta x = \bar{v}_{ax} t + \frac{1}{2} a_x t^2 = \quad \Delta x = +(v_p + v_{m0} \sin(\theta)) t + 0 \quad (Equation B)
\]

\( \Delta x \) could be found from *Equation B* if we knew ‘t’. But, ‘t’ is unknown. i.e., two unknowns in one relationship. However, since \( t_x = t_y \) and we know \( \Delta y \) from *Equation A*, we can find \( t \) from

\[
\Delta y = \bar{v}_{ay} t + \frac{1}{2} a_y t^2 = \quad -(H - h_p) = +(v_{m0} \cos(\theta)) t - \frac{1}{2} g t^2 \quad (Equation C)
\]

Here, \( v_{ax} = (v_p + v_{m0} \sin(\theta)) = 21.0 + 1.95 = 22.95 \frac{\text{m}}{\text{s}} \); and \( v_{ay} = v_{m0} \cos(\theta) = 4.603 \text{ m/s} \)

Rewriting *Equation C* as \( + \frac{1}{2} g t^2 - (v_{m0} \cos(\theta)) t - (H - h_p) = 0 \) and solving the quadratic, we get

\[
t = \frac{+v_{m0} \cos(\theta) \pm \sqrt{(v_{m0} \cos(\theta))^2 + 2g(H - h_p)}}{g} = \frac{+4.603 \pm \sqrt{4.603^2 + 2 \cdot 9.81 \cdot (-168.6)}}{9.81} = 6.35 \text{ s} \quad \text{(only + root gives } t > 0)\]

Negative root leads to a negative time, which is unphysical for ‘time’.

From *Equation A*, we now get

\[
\Delta x = -(H - h_p) = -(22.95) \cdot 6.35 = +145.77 \text{ m}
\]

\[
\Rightarrow |\Delta \vec{r}| = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{145.77^2 + (-168.6)^2} = 222.88 \text{ m}
\]
Problem 4 (20 points)

A river flows due north with a speed of 3.0 m/s and is 700.0 m wide. A man steers a motorboat across the river; his velocity relative to the water is 4.5 m/s and he launches his boat from the west bank heading at an angle of 30 degrees south of east as he crosses the water.

A) What is the velocity of the boat relative to the earth?

B) Where on the east bank of the river will the boat land, relative to the point directly opposite the launch point?

C) How long does it take the boat to cross the river?

\[
\begin{align*}
\text{\textbf{B)} Boat lands on East Bank} \\
\quad d_{\text{NORTH}} &= v_N t = (0.75 \text{ m/s})(180 \text{ s}) = 135 \text{ m NORT}\xbar \\
\quad d_{\text{EAST}} &= 700 \text{ m} = v_E t \\
\quad t &= \frac{700 \text{ m}}{3.89 \text{ m/s}} = 180 \text{ s}
\end{align*}
\]
Problem 5 (20 points)

Suppose you are given a system that is constrained to move in 1-D along the x-axis whose acceleration as a function of time is:

\[ a(t) = K t^4, \text{ where } K \text{ is a known constant.} \]

If at a non-zero time, \( t = t_1 > 0 \), the system is located at \( x(t_1) = 0 \), with a velocity of \( v(t_1) = v_1 \) in the positive x-direction, give expressions for the following in terms of the relevant combination of the known/given parameters ONLY (i.e. \( K, v_1, t, t_1 \) and \( t_2 \)):

A) \( v(t) = \) ?, where \( t \) is any time after \( t_1 \).

B) \( x(t_2) - x(t_1) = \) ?, where \( t_2 \) is some specific time after \( t_1 \).

\[ a(t) = \int a(t) \, dt = \int (K t^4) \, dt = \frac{K}{5} t^5 + C \]

\[ v(t) = \frac{K}{5} t^5 + C = v_1 \quad \Rightarrow \quad C = (v_1 - \frac{K}{5} t_1^5) \]

\[ v(t) = \frac{K}{5} t^5 + (v_1 - \frac{K}{5} t_1^5) \]

\[ x(t) = \int v(t) \, dt = \int [\frac{K}{5} t^5 + (v_1 - \frac{K}{5} t_1^5)] \, dt \]

\[ x(t) = \frac{K}{5} t^6 + (v_1 - \frac{K}{5} t_1^5) t + C \]

\[ t = t_1; \quad x = 0 \]

\[ x(t_1) = 0 = \frac{K}{5} t_1^6 + (v_1 - \frac{K}{5} t_1^5) t_1 + C \]

\[ C = -\frac{K}{5} t_1^6 - (v_1 - \frac{K}{5} t_1^5) t_1 \]

\[ x(t_2) - x(t_1) = \frac{K}{5} (t_2^6 - t_1^6) + (v_1 - \frac{K}{5} t_1^5)(t_2 - t_1) \]