Short Problems:

A. Option (x); B. Option (ii); C. Option (iv); D. Option (iii).

Long Problems:

Problem 2 (20 points)

A man pulls a box of mass $m$ at a constant velocity across a horizontal floor. He applies a force $F$ at an angle of $\theta$ with respect to horizontal (as shown in the figure below). Assume that the rope is massless.

a) Draw a well-labeled free-body diagram for the box.

b) Find the coefficient of kinetic friction $\mu_k$.

c) Find the work done by the force $F$ and work done by the net force after the box is moved horizontally by $\Delta s$.

Your answers MUST only be in terms of the relevant combination of the given quantities ($m, \theta, F, \Delta s$ and $g$) — not all of them may be necessary/applicable in a given part.
Problem 3:

In a conical pendulum, a bob with mass $M = 4 \text{ kg}$ at the end of a thin massless wire of length $L = 250 \text{ cm}$, moves in a horizontal circle at a constant rate of 2 rev/s with the wire making a constant angle with the vertical direction. Calculate -

(a) the net force acting on the bob, and the angle between the wire and the vertical,

(b) what work should be done by friction in order to stop the pendulum.

SOLUTION:

The period is known $T = 1/(2 \text{rev/s}) = 0.5 \text{ s}$ or $T = 1/(4 \text{rev/s}) = 0.25 \text{ s}$. The equations are

(1) The speed $v = 2\pi R/T$, where the radius of the horizontal circle $R = L \sin \beta$ is determined by the angle $\beta$ between the wire and the vertical.

(2) The vertical component of the tension force $T \cos \beta = Mg$.

(3) The net force $T \sin \beta = Mv^2/R$ is responsible for the centripetal acceleration $a_c = v^2/R$.

Answer to question (a): Plugging Eqs. (1) and (2) into Eq. (3), one finds for the angle $\cos \beta = (g/L)T^2/(2\pi)^2$ and for the net force $F = T \sin \beta = Mg \tan \beta$.

Answer to question (b) follows from the work-energy theorem $W = -Mv^2/2 - MgL(1 - \cos \beta)$, where $v^2 = (2\pi L/T)^2 (1 - \cos^2 \beta)$.

$\cos \beta = 0.025, \beta = 88.60^0, F = (1570 \pm 30) \text{ N}, W = -2070 \text{ J}$.

Problem 4: A 500 g block is released from the top of a 45° incline. The block is initially at a distance of $D = 80 \text{ cm}$ from the free end of a long massless spring that lies on the incline’s surface with its other end held fixed at the bottom edge of the incline. The unstretched length of the spring is $L = 17 \text{ cm}$. The block slides down the incline and comes to a momentary stop at the maximum compression of the spring when the spring’s length becomes 13 cm. The spring then rebounds and the box begins to move back up the plane. The static and kinetic coefficients of friction between the block and the incline are equal $\mu_k = \mu_s = 0.6$. Find the total energy loss due to friction over the entire way from the initial location of release all the way to the highest position the block attains after the rebound.

SOLUTION:

The friction work is $W = -\mu_k Mg(2d-x)\cos(45^0)$, where $d = (D+L-L_1)$ with $L_1$ being the compressed spring’s length. Due to work-energy theorem $Mg\times(45^0) = \mu_k Mg(2d-x)\cos(45^0)$, one finds a distance from the initial position to the highest position after rebound: $x = 2\mu_k d/(1+\mu_k)$. The energy loss is $W = -2^{1/2} Mg \mu_k/(1+\mu_k)$. This solution is valid if $x < d$ (that is easy to check).

$W = -2.2 \text{ J}$
Problem 5 (20 points)

An elementary particle (electron) of mass $m_e$ is launched from the origin $(x, y, z) = (0,0,0)$ of a certain coordinate system with an initial speed $v_0$ and is thereafter confined in space under the interaction with a spherically symmetric external field. The potential energy of the electron-field system is described by the function $U(x, y, z) = A(x^2 + y^2 + z^2)$, where $A$ is a positive constant. The electron is not interacting with anything else.

a) Draw an energy diagram for the electron as a function of distance ($r$) from the origin. Identify the potential and kinetic energy at a distance $r_1$ on the plot.

b) What is the maximum distance $r_{\text{max}}$ that this electron can move away from the origin? Your answer MUST be in terms of the relevant combination of the known/given quantities $A, v_0$, and the mass of an electron $m_e$ ONLY; not all may be necessary.

c) Find the acceleration vector $\vec{a}$ of this electron when it is located at a point described by the coordinates $(x_2, y_2, z_2)$. Assume that $(x_2, y_2, z_2)$ is closer to the origin than the distance $r_{\text{max}}$ found in part b).

8) Use energy conservation (total mechanical energy of the syst. is conserved)

At origin $E_{\text{mech}} = K; \quad u(0,0,0) = 0$

At $r_{\text{max}}$  $K = 0; \quad E_{\text{mech}} = U = A r_{\text{max}}^2$

\[
\frac{1}{2} m_e v_0^2 = A \left( \frac{m_e v_0^2}{2A} \right) = \sqrt{\frac{m_e v_0^2}{2A}}
\]

\[
\Gamma_{\text{max}} = \frac{m_e v_0^2}{2A} = \sqrt{\frac{m_e v_0^2}{2A}}
\]

C) Use 2nd Newton's Law $\vec{F} = m \vec{a}$

force is given by potential energy

\[
\vec{F} = -\left( \frac{\partial U}{\partial x} \hat{i} + \frac{\partial U}{\partial y} \hat{j} + \frac{\partial U}{\partial z} \hat{k} \right)
\]

\[
\vec{a} = -\frac{2}{m_e} A \left( x_2 \hat{i} + y_2 \hat{j} + z_2 \hat{k} \right)
\]

at $(x_2, y_2, z_2)$