Electromagnetic Induction

Chapter 29

November 1, 2012
Exam 3 Review Session

- I will hold a review for Exam 3 which covers Chapters 27, 28, 29 and 30, on Wednesday November 7\textsuperscript{th} at 7:15pm in MPH Y 205.
- Exam 3 will be given in class on Thursday, November 8\textsuperscript{th}.
Learning Goals for Chapter 29

- The experimental evidence that a changing magnetic field induces an EMF.
- How Faraday’s Law relates the induced EMF in a loop to the change of magnetic flux through the loop.
- How to determine the direction of the induced EMF.
- How to calculate the EMF induced in a conductor moving through a magnetic field.
- How a changing magnetic flux generates an electric field that is very different from that produced by an arrangement of charges.
- The four fundamental equations that completely describe both electricity and magnetism. (Maxwell’s Equations)
Faraday's Law

\[ \mathcal{E}_{\text{INDUCED}} = -\frac{d}{dt} \Phi_B = -\frac{d}{dt} (\mathbf{B} \cdot d\mathbf{A}) \]

where

\[ \Phi_B = \int \mathbf{B} \cdot d\mathbf{A} \]

through the loop which you wish to find the induced EMF around.
Faraday’s Law……

- The direction of the induced EMF is given by the following rule:
  - Define a positive direction for the area vector \( \mathbf{A} \).
  - From the direction of \( \mathbf{A} \) and the magnetic field \( \mathbf{B} \) determine the sign of the magnetic flux and its rate of change with time.
  - Determine the sign of the induced EMF, if the flux is increasing the rate of change of flux is positive.
  - Finally, determine the direction of the induced EMF using the Right Hand Rule by curling the fingers of your right hand around the vector \( \mathbf{A} \). If the induced EMF is positive, it is in the same direction as the curled fingers and if it is negative, it is in the opposite direction.
Lenz’s Law on the direction of these induced currents....

The direction of the induced current is such that the induced current it causes to flow will produce a magnetic field which tends to counter act the “CHANGE IN FLUX” through the circuit.
EMF’s when moving conductors through a B-field

\[ \text{Magnetic Force} = q \vec{v} \times \vec{B} \]

Charge moves in the conductor as long as the force on the charges is non-zero. For moving bar positive charge wave to one end and negative to the other, creating an electric field such that \( \vec{E} \propto \vec{v} \times \vec{B} \).
Calculating these motional EMF’s

We can then write down the EMF for the induced E-field.

\[ \varepsilon_{\text{induced}} = \int \vec{E} \cdot d\vec{l} = -\int_{t_0}^{t_1} \frac{\partial \vec{B}}{\partial t} \cdot d\vec{l} \]

So for the moving bar in our example,

\[ \varepsilon_{\text{induced}} = \int_0^L (\vec{v} \times \vec{B}) \cdot d\vec{l} = \int_0^L v \vec{B} \cdot d\vec{l} = vBL \]

With the end of the bar coming out of the page being at the higher potential.
EMF’s without wires…. 

Indeed E-fields in regions of space without wires present ….

we find that in “space” where the B-field may be changing with time we can also have E-fields induced even in the absence of conducting to form a circuit!
We use a loop of wire to calculate the induced EMF here but we find that the EMF can exist with only the required path to integrate around....

November 1, 2012
Eddy Current's

A rotating Conducting disk

Drag force resulting from the ILB force on these currents

Induced currents produced in the moving conductor..
The Displacement Current ....

What happens to the magnetic field in the following situation.

\[ \int \mathbf{B} \cdot d\mathbf{l} \quad \text{Surface to calculate } \mathbf{B} \cdot d\mathbf{l} \]

Far from the capacitor, the field should look like that from a long wire. However, look what happens to Ampère's Law ....

The integral of \( \mathbf{B} \cdot d\mathbf{l} \) around the path should be the same as before. But through the balloon shaped surface is zero??
As the current flows... charging the capacitor, we know the E-field in the region is changing.

\[ E(\text{at some charge } q) = \frac{q}{\varepsilon_0} \]

so \[ \frac{dE}{dt} = \frac{1}{A\varepsilon_0} \frac{d\Phi}{dt} = \frac{I}{A\varepsilon_0} \]

If we consider the I_{eff} in the capacitor, we have

\[ I_{eff} = A\varepsilon_0 \frac{dE}{dt} \]

\[ = \varepsilon_0 \frac{dE}{dt} \cdot A = \varepsilon_0 \frac{d\Phi}{dt} \]

so between the plates we have the effective current producing the B-field. To take this into account, Ampere's law becomes

\[ \oint \mathbf{B} \cdot d\mathbf{e} = \mu_0 (I_{\text{rem}} + \varepsilon_0 \frac{d\Phi}{dt}) \]

Corrected Ampere’s Law for Changing Electric Fields.
Magnetic field inside a charging capacitor

Inside the capacitor

\[ \oint \mathbf{B} \cdot d\mathbf{l} = B(2\pi r) = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \]

\[ B(r) = \frac{\mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}}{2\pi} = \frac{\mu_0 \varepsilon_0}{2} r \frac{dE}{dt} \]
MAXWELL'S

EQUATIONS

These summarize ALL of the principles of electricity & magnetism in just 4 relationships ......

- $\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$
- $\oint \mathbf{B} \cdot d\mathbf{A} = 0$
- $\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \Phi_{\text{magnetic}}$
- $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 (I + \varepsilon_0 \frac{d}{dt} \Phi_{\text{electric}})$

We will return to these equations in a couple of weeks when we get to our discussion of EM waves ....
Inductance

Chapter 30

November 1, 2012
Learning goals for Chapter on Inductance

- How a time varying current in one coil can induce an EMF in a second coil.
- How to relate the induced EMF in a circuit to the rate of change of the current in that circuit.
- How to calculate the energy stored in a magnetic field.
- How to analyze circuits that include both a resistor and an inductor.
- Why electrical oscillations occur in circuits that include both an inductor and a capacitor.
- Why oscillations decay in circuits with an inductor, resistor and capacitor.
A new circuit element based on Faraday’s Law

INDUCTANCE

Discussion of the circuit elements which operate based on Faraday’s Law of Induction.

An inductor is the name we gave to this class of circuit elements. In our discussion we will see that any device which produces a magnetic field can be an inductor.
A new circuit element based on Faraday’s Law
Two kinds of inductance....

Circuit 1

The current flowing in Circuit 1 produces a magnetic field. If the current should change with time, then this field will also change with time.

Circuit 2

If $B_1(t)$ changes with time, the flux through Loop 2 changes too.
The EMF induced in circuit 2 due to circuit 1

Using Faraday's law we know that an EMF will be produced in circuit 2

\[ E_{\text{ind}2} = -\frac{d\Phi_2}{dt} = -N_2 \frac{d\Phi_1}{dt} \]

If we define \[ \Phi_2 = N_2 \Phi \]

then

\[ E_{\text{ind}2} = -N_2 \frac{d\Phi}{dt} \]

Thus, constant \( M_{21} \) is called the mutual inductance and \( \Phi \) is \( \Phi_1 \) due to \( \Phi \).
The Mutual Inductance $M_{21}$ is a constant which depends only on the relative geometry of the two circuits and the space connecting them. It does not depend on the current flowing.

**Units of Mutual Inductance**

$$M = \frac{\text{Volts}}{\text{Amp/Sec}} = \text{Henry}$$
Example…

- Suppose we see an EMF of 5.0 volts in circuit 2 due to a changing current in circuit 1.
- Suppose the current in circuit 1 is changing at 4.0 Amps/sec.
- What is the Mutual inductance of these circuits??  \( M = \frac{\text{EMF}_2}{\frac{\text{dI}_1}{\text{dt}}} \)
Now for the EMF induced in circuit 1 due to circuit 2

\[
\text{EMF}_1 = \frac{d\Phi_1^\text{TOT}}{dt} = -N_1 \frac{d\Phi_1^\text{ONE LOOP}}{dt}
\]

with \( \Phi_1^\text{TOT} = N_1 \Phi_1^\text{ONE LOOP} = M_{12} i_2 \)

Also,

\[
\text{EMF}_1 = -M_{12} \frac{d i_2}{dt}
\]
The symmetry between the two circuits...

While it is not so easy to prove for all cases, it turns out that there is a connection between $N_{21}$ and $M_{12}$ in a system

$M_{12} = M_{21} = \text{Mutual Inductance}$

with

$N_1 = \frac{N_2 \phi_2}{i_1} = \frac{N_1 \phi_1}{i_2}$

$\text{Eur. induced} = -H \frac{d \text{Flux}}{dt}$
Another example of Mutual Inductance...

EXAMPLE: calculating inductances of a system of loops.

\[ B \]

VERY LONG SOLENOID

\[ N \text{ wires/meter} \]

Carrying current \( i \) at

\[ r \]

\[ N \] turns of wire of radius \( R \)
Finding the Mutual Inductance

To find the mutual inductance of this system, we first calculate the flux through the large loop due to the solenoid.

\[ \Phi = \int \vec{B} \cdot d\vec{A} = \int (\mu_0 n_1 i) dA - n_1 i \int dA \text{ solenoid} \]

\[ = (\mu_0 n_1 i) \pi a^2 \]

area of solenoid

There is no B-field outside. No matter how large a loop we close, the solenoid

Then,

\[ M = \frac{N J \mu_0 i (\pi a^2)}{\gamma} = \frac{N \mu_0 \pi a^2}{\gamma} \]
Filling some numbers to get a feeling for the size of this effect...

If in our example

\[ N = 100 \text{ turns} \]

and \( a = 10 \text{ cm} \)

\[ M = \left( 4\pi \times 10^{-7} \right) \left( 100 \right) (1000 \text{ m}^{-1}) (\pi \times 10^{-2} \text{ m}^2) \]

\[ = 3.94 \times 10^{-3} \text{ Henrys} = 3940 \mu \text{H} \]

**NOTE:** Typical values of inductances range from a few Henries down to microHenries....

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