Electric Potentials

Chapter 23

September 18, 2012
Reminders

- Exam 1 over chapters 21, 22 and 23 will be in class Thursday, 9/20/12.
- I will hold a review session on Wednesday afternoon 9/19/12 from 5:30-7:00 pm in room MPH 204 for those interested.
- Sample exams are available on the class web site. http://faculty.physics.tamu.edu/webb/208_RCW/WEBB.HTM
- Formula sheets will be provided with the exam on Thursday.
Coulomb Potential Energy

\[ U_{\text{Coulomb}} (r) = \frac{kQq}{r} + \text{constant} \]

This constant is completely arbitrary, however, our convention is to set it to zero when charges are infinitely far apart.

This \( r \) is the distance between the two point charges.
The Potential Function
from a slightly different point of view.

If we take and divide the work done by the Coulomb Force by the charge being moved around we get an expression relating the Electric field to a new quantity... The Electric Potential...

\[
\frac{W_{\text{Force}}^{A\rightarrow B}}{q} = \int_{A}^{B} \frac{\vec{F} \cdot d\vec{l}}{q} = \int_{A}^{B} \frac{\vec{E} \cdot d\vec{l}}{q} = \int \vec{E} \cdot d\vec{l}
\]

this work per unit charge is equal to the difference in a New function.

\[
\frac{W_{\text{Force}}^{A\rightarrow B}}{q} = -\frac{\Delta U}{q} = -\Delta V \quad \text{so...}
\]

\[-\int \vec{E} \cdot d\vec{l} = \Delta V
\]

this is one of the ways of defining the potential function.
Now for a collection of point charges....

\[ Q_2 = +50 \, \mu C \]

\[ Q_1 = -50 \, \mu C \]
The calculation...

To find the potential due to these two charges we can use the earlier definition of the voltage difference.

\[ \Delta V_{\text{total}} = V(r) - V(\infty) = -\int \vec{E}_{\text{total}} \cdot d\ell \]

with \( \vec{E}_{\text{total}} = \vec{E}_1 + \vec{E}_2 \)

solving for \( \Delta V_{\text{total}} \) we get the following..

\[ \Delta V_{\text{total}} = \int_{\infty}^{r} \frac{kq_1}{r_1^2} \hat{n} \cdot d\ell - \int_{\infty}^{r} \frac{kq_2}{r_2^2} \hat{r}_2 \cdot d\ell = \frac{kq_1}{r_1} \bigg|_{\infty}^{r} + \frac{kq_2}{r_2} \bigg|_{\infty}^{r} = \]

\[ = \Delta V_1 + \Delta V_2 \] the total potential is just the sum of the two potentials!!
Finishing the calculation..

\[ V_{total} = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} \] so to find the potential at point B we have..

\[ V_{total} = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} \] with

\[ r_1 = \sqrt{(x - x_1)^2 + (y - y_1)^2} \] and \[ r_2 = \sqrt{(x - x_2)^2 + (y - y_2)^2} \]

and \( x = 0 \) cm and \( y = 36 \) cm.
The results..

Plugging in the numbers we get the following.

\[ V_{total} \text{ (at point B)} = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} = \]
\[ = \left(9 \times 10^{-9}\right) \frac{(-50 \mu C)}{\sqrt{(0.26)^2 + (0.30)^2}} + \left(9 \times 10^{-9}\right) \frac{(50 \mu C)}{\sqrt{(0.26)^2 + (0.30)^2}} \]
\[ = 0 \text{ volts} \]

\[ V_{total} \text{ (at point A)} = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} = \]
\[ = \left(9 \times 10^{-9}\right) \frac{(-50 \mu C)}{\sqrt{(0.52)^2 + (0.30)^2}} + \left(9 \times 10^{-9}\right) \frac{(50 \mu C)}{\sqrt{(0.52)^2 + (0.30)^2}} \]
\[ = 7.5 \times 10^{-5} \text{ volts} \]
Now for continuous charge distributions..

- We saw for a collection of point charges that the total potential is just the sum of the individual potentials, then to find the potential for a continuous distribution requires that we ADD together the contributions for each point charge element of the distribution.
The integral form for this sum.

$$V_{total} = \int dq' \frac{k dq'}{|\vec{r} - \vec{r}'|}$$

Location where you wish to find the potential.

The charge producing the potential.

NOTE: all quantities are scalars!

Location of the charge producing the potential.
Finding the potential for a continuous charge distribution.

- Identify the relevant charge element \( dq' \) (volume, surface, line).
- Using the distance formula, calculate the distance between the charge and the point of interest.
- Plug a) and b) into the integral and integrate.

\[
V_{total} = \int dq' \frac{k dq'}{|\vec{r} - \vec{r}'|}
\]
Example 1..

Find the potential along the axis of a uniform ring of charge of radius $R_0$ and total charge $Q$. 

\[
\frac{dq}{(x^2 + R^2)^{\frac{3}{2}}}
\]
Following the steps:

\[ dq' = \lambda d\ell' = \lambda R_o d\theta' \text{ and } \]

\[ |\vec{r} - \vec{r}'| = \sqrt{R_o^2 + z^2} \text{ so...} \]

\[ V_{\text{total}} = \int_{dq'} \frac{k\lambda R_o d\theta'}{\sqrt{R_o^2 + z^2}} = \frac{k\lambda R_o}{\sqrt{R_o^2 + z^2}} \int d\theta' = \]

\[ V_{\text{total}} = \frac{k\lambda R_o 2\pi}{\sqrt{R_o^2 + z^2}} + V_{\text{ref}} \]
Example 2....

Let’s find the potential due to this uniform line charge segment along a) the x axis and b) along the y axis.
Going through the steps...

\[ dq' = \lambda d\ell' = \lambda dx' \] and

\[ |\vec{r} - \vec{r}'| = \sqrt{(x - x')^2} \] for the x axis and...

\[ |\vec{r} - \vec{r}'| = \sqrt{(-x')^2 + y^2} \] for points along the y axis....

plugging this in gives.....

\[ V_{total} = \int \frac{k\lambda dx'}{dq'} \sqrt{(x - x')^2} = -k\lambda \ell n\left(\frac{x - x'}{x - \ell}\right) \] for the x axis

\[ V_{total} = \int \frac{k\lambda dx'}{dq'} \sqrt{(-x')^2 + y^2} = -k\lambda \ell n\left(\frac{-x'}{(-x')^2 + y^2}\right) = k\lambda \ell n\left(\frac{(\ell) + \sqrt{(\ell)^2 + y^2}}{(-\ell) + \sqrt{(-\ell)^2 + y^2}}\right) \] for the y axis...
Using $V(r)$ to find $E(r)$

- One of the benefits of using potentials instead of E-fields is that we can retrieve the E-field information from these functions, since...

$$- \int \vec{E} \cdot d\vec{l} = \Delta V$$
Differentiate both sides with respect to $dl$.

\[-\int \vec{E} \cdot d\vec{\ell} = \Delta V\]

differentiating both sides with respect to $d\vec{\ell}$ gives

the component of $E$ along $\ell$, $E_\ell = -\frac{dV}{d\ell}$.
In component form...

\[ E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z} \]

where the \( \partial x \) represents the partial derivative with respect to \( x \) and so on for the other components.
Partial derivatives....

To calculate a partial derivative with respect to a particular variable, you treat ALL OTHER variables as CONSTANTS. .......

If $V(x, y) = x^2 + y^2 + 2xy$ then ..... 

\[
\frac{\partial V}{\partial x} = 2x + 2y \quad \text{and} \quad \frac{\partial V}{\partial y} = 2y + 2x
\]
Let’s find E for a line segment charge distribution...

Remember from the earlier problem we found...

\[ V_{\text{total}} = k\lambda \ln \left\{ \frac{x + \ell}{x - \ell} \right\} \] for points on the x axis and...

\[ V_{\text{total}} = k\lambda \ln \left\{ \frac{(\ell) + \sqrt{(\ell)^2 + y^2}}{(-\ell) + \sqrt{(-\ell)^2 + y^2}} \right\} \] for points on the y axis...
Let's look at the x-axis first..

\[ V_{total} = k \lambda \ln \left( \frac{x + \ell}{x - \ell} \right) \]

for points on the x axis then taking the partial derivatives with respect to x we get the following...

\[ E_x = -\frac{\partial V}{\partial x} = -k \lambda \left\{ \frac{1}{x + \ell} - \frac{1}{x - \ell} \right\} \]

which is what we saw previously in the homework from chapter 21...
Now for the y-axis..

\[ V_{\text{total}} = k\lambda\ell \ln \left\{ \frac{(\ell) + \sqrt{(\ell)^2 + y^2}}{(-\ell) + \sqrt{(-\ell)^2 + y^2}} \right\} \text{ for points on the y axis} \]

now taking the partial derivative with respect to y we get.....

\[ E_y = -k\lambda \left\{ \frac{1}{\ell + \sqrt{\ell^2 + y^2}} \left( \frac{1}{2} \left( \frac{1}{\ell^2 + y^2} \right) \frac{1}{2} 2y \right) \right\} - \]

\[ \left\{ \frac{1}{-\ell + \sqrt{\ell^2 + y^2}} \left( \frac{1}{2} \left( \frac{1}{\ell^2 + y^2} \right) \frac{1}{2} 2y \right) \right\} \]

doing the algebra... ...
Continuing…

\[ E_y = -k \lambda \left\{ \frac{1}{\ell + \sqrt{\ell^2 + y^2}} - \frac{1}{-\ell + \sqrt{\ell^2 + y^2}} \right\} \frac{y}{\left(\ell^2 + y^2\right)^{1/2}} = \]

\[ = -k \lambda \left\{ \frac{-2\ell}{y^2} \right\} \left( \frac{y}{\left(\ell^2 + y^2\right)^{1/2}} \right) = \frac{2k \lambda}{y} \left( \frac{\ell}{\left(\ell^2 + y^2\right)^{1/2}} \right) \]

which is what we had calculated previously...
Recap of Chapter 21
Coulomb’s Law

• The nature of electric charge and its conservation.
• How objects become “charged”.
• How to use Coulomb’s Law to calculate the electric forces between charges.
• Understand the difference between electric force and electric field.
• How to calculate the electric field due to a collection of point charges.
• How to use the concept of electric field to visualize and interpret electric fields.
• How to calculate the properties of electric dipoles.
Recap of Chapter 21
Coulomb’s Law

Coulomb’s law

Electric field [N/C = V/m]  (point charge q)

Electric force [N]  (on q in E)

\[ |\vec{F}| = k \frac{|q_1| |q_2|}{r^2} \]
\[ \vec{E}(r) = \vec{k} \frac{q}{r^2} \hat{r} \]
\[ (\hat{r} = \text{unit vector radially from } q) \]
\[ \vec{E} = \sum \vec{E}_i = k \sum \frac{q_i}{r_i^2} \hat{r}_i \]
\[ \vec{E} = k \int \frac{dq}{r^2} \hat{r} \]
\[ (\hat{r} = \text{unit vector radially from } dq) \]
\[ \vec{F} = q \vec{E} \]
Recap of Chapter 22
Gauss’s Law

• How you can determine the amount of charge within a closed surface by examining the electric field on the surface?

• What is meant by electric flux and how to calculate it?

• How Gauss’s Law relates the electric flux through a closed surface to the charge enclosed by the surface?

• How to use Gauss’s Law to calculate the electric field due to symmetric charge distribution?

• Where the charge is located on a charged conductor?
Recap of Chapter 22

Gauss’s Law

Electric flux  (through a small area $\Delta A_i$)  \[ \Delta \Phi_i = \vec{E}_i \cdot \Delta \vec{A}_i = E_i \, \Delta A_i \cos \theta_i \]

(through an entire surface area)  \[ \Phi_{\text{surface}} = \lim_{\Delta A \to 0} \sum \Delta \Phi_i = \int \vec{E} \cdot d\vec{A} \]

Gauss’ law  (through a closed surface area)  \[ \Phi_{\text{closed}} \equiv \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\varepsilon_0} \]
Recap of Chapter 23

Electric Potentials

• How to calculate the electric potential energy of a collection of charges.

• The meaning and significance of the electric potential.

• How to calculate the electric potential for a collection of point charges at a point in space.

• How to use equipotential surfaces to visualize how the electric potential varies in space.

• How to use the electric potential to calculate the electric field.
Recap of Chapter 23
Electric Potentials

Electric potential \([ V = J/C ]\) (definition)
- \( \vec{E} = \text{constant} \)
- (point charge \( q \))
- (group of charges)
- (continuous charge distribution)

\[
\Delta V = V_B - V_A = - \int_A^B \vec{E} \cdot d\vec{s}
\]

\[
\Delta V = -\vec{E} \cdot (\vec{r}_B - \vec{r}_A)
\]

\[
V(r) = k \frac{q}{r} \quad \text{(with } V(\infty) = 0)\]

\[
V(\vec{r}) = \sum_{i=1} V_i(|\vec{r}_i - \vec{r}|) = k \sum \frac{q_i}{|\vec{r}_i - \vec{r}|}
\]

\[
V(\vec{r}) = k \int \frac{dq}{|\vec{r}' - \vec{r}|} \quad \text{(with } V(\infty) = 0)\]

Electric potential energy \([ J ]\) (definition)

Electric potential energy of two-charge system

\[
\Delta U = U_B - U_A = -q_0 \int_A^B \vec{E} \cdot d\vec{s}
\]

\[
= q_0 (V_B - V_A)
\]

\[
\vec{E} = -\nabla V \quad \text{(gradient operator)}
\]

\[
U_{12} = k \frac{q_1 q_2}{r_{12}}
\]