E-field for a negative charge

Region of uniform E-field

Region of non-uniform E-field

Field gets weaker in this direction

Field gets stronger this way
Field lines start on + charges and end on – charges......
Field lines cannot cross...
Density of lines is proportional to the strength of the field....
Chapter 22
Gauss’s Law
September 6, 2012
Chapter 22 Learning Goals

- How you can determine the amount of charge within a closed surface by examining the electric field on the surface?
- What is meant by electric flux and how to calculate it?
- How Gauss’s Law relates the electric flux through a closed surface to the charge enclosed by the surface?
- How to use Gauss’s Law to calculate the electric field due to symmetric charge distribution?
- Where the charge is located on a charged conductor?
Flux through a surface...

\[ \Phi = \oint d\Phi = \oint \vec{E} \cdot d\vec{A} \]

Open surface

Closed surface
Estimating flux calculations

- We can take advantage of the fact that the flux value is proportional to the number of field lines passing through a surface.

- We must use an E-field map that has been drawn following our rules for field lines.
Sample calculations.....
What pattern is emerging here?

- If there is no charge \textit{inside} the surface the flux is always zero, no matter what the shape of the surface.
- When there is charge inside, the flux has a value that is always some multiple of the charge enclosed.
- How can we use this info to find the electric field produced by a distribution of charge??
Let’s see if we can come up with a relationship between the flux and charge..

- Let’s calculate the flux from a point charge..

\[
\Phi = \oint \vec{E} \cdot d\vec{A} = \oint \left( k \frac{q}{r^2} \hat{r} \right) \cdot (dA \hat{r})
\]

\[
= k \frac{q}{r^2} \cdot 4\pi r^2 = kq(4\pi)
\]

\[
= \frac{q}{\varepsilon_0}
\]
Gauss’s Law

- The result that we just calculated for a point charge at the center of our surface turns out to be true for any surface and any distribution of charge!!

\[ \Phi = \int \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0} \]
More on Gauss’s Law

- The flux calculation ONLY depends on the charge contained inside the Gaussian surface, but the field whose flux we are calculating is being produced by ALL the charges present.
- Gauss’s Law works for any surface and charge distribution!!
- We can use Gauss’s Law to calculate fluxes through a given surface provided we know where the charges are.
Problem-Solving Strategy 22.1 Gauss’s Law

**IDENTIFY** the relevant concepts: Gauss’s law is most useful when the charge distribution has spherical, cylindrical, or planar symmetry. In these cases the symmetry determines the direction of $\vec{E}$. Then Gauss’s law yields the magnitude of $\vec{E}$ if we are given the charge distribution, and vice versa. In either case, begin the analysis by asking the question: What is the symmetry?

**SET UP** the problem using the following steps:
1. List the known and unknown quantities and identify the target variable.
2. Select the appropriate closed, imaginary Gaussian surface. For spherical symmetry, use a concentric spherical surface. For cylindrical symmetry, use a coaxial cylindrical surface with flat ends perpendicular to the axis of symmetry (like a soup can). For planar symmetry, use a cylindrical surface (like a tuna can) with its flat ends parallel to the plane.

**EXECUTE** the solution as follows:
1. Determine the appropriate size and placement of your Gaussian surface. To evaluate the field magnitude at a particular point, the surface must include that point. It may help to place one end of a can-shaped surface within a conductor, where $\vec{E}$ and therefore $\Phi$ are zero, or to place its ends equidistant from a charged plane.
2. Evaluate the integral $\oint \vec{E}_\perp \, dA$ in Eq. (22.9). In this equation $E_\perp$ is the perpendicular component of the total electric field at each point on the Gaussian surface. A well-chosen Gaussian surface should make integration trivial or unnecessary. If the surface comprises several separate surfaces, such as the sides and ends of a cylinder, the integral $\oint E_\perp \, dA$ over the entire closed surface is the sum of the integrals $\int E_\perp \, dA$ over the separate surfaces. Consider points 3–5 as you work.
3. If $\vec{E}$ is perpendicular (normal) at every point to a surface with area $A$, if it points outward from the interior of the surface, and if it has the same magnitude at every point on the surface, then $E_\perp = E = \text{constant}$, and $\int E_\perp \, dA$ over that surface is equal to $EA$. (If $\vec{E}$ is inward, then $E_\perp = -E$ and $\int E_\perp \, dA = -EA$.) This should be the case for part or all of your Gaussian surface. If $\vec{E}$ is tangent to a surface at every point, then $E_\perp = 0$ and the integral over that surface is zero. This may be the case for parts of a cylindrical Gaussian surface. If $\vec{E} = 0$ at every point on a surface, the integral is zero.
4. Even when there is no charge within a Gaussian surface, the field at any given point on the surface is not necessarily zero. In that case, however, the total electric flux through the surface is always zero.
5. The flux integral $\oint E_\perp \, dA$ can be approximated as the difference between the numbers of electric lines of force leaving and entering the Gaussian surface. In this sense the flux gives the sign of the enclosed charge, but is only proportional to it; zero flux corresponds to zero enclosed charge.
6. Once you have evaluated $\oint E_\perp \, dA$, use Eq. (22.9) to solve for your target variable.

**EVALUATE** your answer: If your result is a function that describes how the magnitude of the electric field varies with position, ensure that it makes sense.
Use Gauss’s’s Law to find flux.
Example calculations...

- Suppose we have a point charge of 3nC surrounded by and at the center of a spherical surface of radius, 1m. What is the flux of the electric field through this surface?
Example calculation continued

- Now, take the same charge and offset it 10 cm from the center of this sphere. What is the flux now through this surface?
Examples continued

- Lastly, suppose there are two point charges inside the sphere. One of 3 nC and the other of -2 nC. What is the flux of the e-field through the surface under these conditions?
Using Gauss’s Law to find $E$..

- Another feature of Gauss’s Law is that for certain geometries of charge distributions, we can use Gauss’s Law to find the E-fields due to a given set of charges.

  - This works for a limited set of geometries
    - Spherically symmetric distributions
    - Infinite line charges or cylindrical distributions
    - Infinite sheet charges or slabs of charge
Spherically symmetric distributions

Gaussian Surface
Long(infinite) cylindrical distributions

Gaussian Surface
Infinite sheets/slabs of charge

Gaussian Surface

\[ E \]

\[ A \]

\[ d \]

\[ x \]

\[ y \]
Using Gauss’s Law to find $E$

- Identify the symmetry of the charge distribution.
- Select appropriate Gaussian Surface.
- Use symmetry to determine the direction of the $E$-field.
- Evaluate the "total flux" through the surface chosen.
- Calculate the enclosed charge.
- Use Gauss’s Law and solve for the $E$-field.
Using Gauss’s Law...an example

- Let’s find the Electric field produced by a uniformly charged “insulating sphere” of radius R and charge density rho.

  - There are two regions of interest here.
    - \( r < R \) inside the distribution.
    - \( r > R \) outside the distribution.

\[
\Phi = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}
\]
For $r$ inside the distribution, only the charge inside the Gaussian Surface is to be included.

\[
q_{\text{enclosed}} = \int \rho_o \, dv = \rho_o \left(\frac{4}{3}\right) \pi r^3
\]
Sooooo (r < R cont.)..

Now setting the integral over the surface of the flux to the enclosed charge we get...

\[ E \ 4\pi \ r^2 = \frac{q_{\text{enclosed}}}{\varepsilon_0} = \frac{\rho_o \left(\frac{4}{3}\right)\pi r^3}{\varepsilon_0} \]

Solving for E gives.....

\[ E = \frac{\rho_o \left(\frac{4}{3}\right)\pi r^3}{4\pi r^2} = \frac{\rho_o}{3\varepsilon_0} r \text{ radially outward} \]
For $r > R$ we get...

$$q_{\text{enclosed}} = \int \rho_o dv = \rho_o \left( \frac{4}{3} \right) \pi R^3 = Q_{\text{total}}$$

$$E \ 4\pi \ r^2 = \frac{q_{\text{enclosed}}}{\varepsilon_o}$$

or solving for $E = \frac{Q_{\text{total}}}{\varepsilon_o \ 4\pi \ r^2}$ radially outward

this is just the field of a point charge!!!!