Series resistor circuits...

In series circuits, the same current flows through all the components in the circuit; the total voltage across the series elements is the sum of the voltages across each of the elements.
Parallel resistor circuits...

- In a parallel circuit, all components have the same voltage across them; the total current delivered is the sum of the currents through each of the elements.
Now for series network of resistors

Using the definition for \( R \).

\[
R_{\text{total}} = \frac{\Delta V_{\text{total}}}{I_{\text{total}}} \quad \text{with}
\]
\[
V_{\text{total}} = V_1 + V_2 + V_3 \quad \text{then}
\]
\[
R_{\text{total}} = \left( I_1 R_1 + I_2 R_2 + I_3 R_3 \right) / I_t
\]

then

\[
R_{\text{total}} = R_1 + R_2 + R_3.
\]

They just add together…

October 9, 2012

Physics 208
Parallel networks of resistors..

- Here, using the definition for $R_{\text{total}}$ we find that $R_{\text{total}} = \frac{\Delta V_{\text{total}}}{I_{\text{total}}}$. With $I_{\text{total}} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$ then

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}...$$
Combination circuits.

(a)

(b)

A

R

R

R

B

R

R

R
Multiloop circuits and Kirchhoff’s Rules

Kirchhoff’s Rules are based on two conservation laws that we have seen before in different a context.

- Conservation of Charge &
- Conservation of Energy
Rule # 1...the Junction Rule

- The current entering ANY junction in a circuit equals the current leaving that same junction.
Rule # 2…the Loop Rule

- The sum of the changes in potential around ANY Closed path in a circuit must be zero
Our conventions...

In solving multiloop problems we use the following **sign conventions** for labeling potential differences.

- For resistors, their potential difference is **negative** if your loop direction and current direction are the same and it is **positive** if your loop direction and current direction are opposite.

- For batteries, their potential difference is **positive** if the direction of the loop is from the negative terminal toward the positive and it is **negative** if the loop direction is from the positive terminal toward the negative.
Multiloop circuit

Problem solving strategy

- Label terminals of the battery.
- Choose the direction of the currents you will be using and label them clearly. (Note: this selection can be done completely arbitrarily. If the direction of the “real” current is opposite the direction you have selected, your answer will be negative.)
- Apply Kirchhoff’s Junction Rule at one or more junctions in the circuit.
- Choose the paths and directions for the loops that you will consider in the circuit.
- Apply Kirchhoff’s Loop Rule around one or more of these loops, remembering to follow the conventions we have adopted.
- Solve the equations algebraically for the unknowns.
A sample problem

\[ V_1 = 9.0 \, \text{V} \quad R_1 = 22 \, \Omega \]

\[ V_3 = 6.0 \, \text{V} \]

\[ R_2 = 15 \, \Omega \]
Setting up the problem

- Junction Rule application
  - \( I_1 = I_2 + I_3 \)

- Loop 1
  - \( 9 - I_3(15 \, \Omega) + I_1(22 \, \Omega) = 0 \)

- Loop 2
  - \( 6 + I_3(15 \, \Omega) = 0 \)
Measuring Voltage and Currents

Ideal vs Real meters

(a) Ideal meter

(b) Real meter

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We will deal with a simple series RC circuit and look at how the currents and voltages in this circuit change with time.
Let’s consider charging the capacitor first…

- Using the definition of capacitance, where \( C = \frac{Q}{\Delta V} \) and
- Kirchhoff’s Loop Rule

We can write down an expression for the voltage drops around this simple RC circuit in the following form….

\[ V_{\text{battery}} - I R - \frac{Q}{C} = 0 \quad \text{where} \quad I = \frac{dQ}{dt} \]
The final form of this loop equation is as follows:

$$V_{\text{battery}} - \left(\frac{dQ}{dt}\right) R - \frac{Q}{C} = 0$$

This is a “linear, first order, differential equation”!! But don’t despair just yet, we don’t need DE to solve this!!
Solving for $Q(t)$.....

starting from...

\[ V_{battery} - R \frac{dQ(t)}{dt} - \frac{Q(t)}{C} = 0 \]

We can do some algebra to rewrite this as....

\[ \frac{dQ(t)}{dt} = \frac{V_{battery} C - Q(t)}{RC} \]

one last bit of algebra and then we can integrate. ...

\[ \frac{dQ(t)}{Q(t) - V_{battery} C} = \frac{-dt}{RC} \]
\[
\frac{dQ(t)}{Q(t) - V_{\text{battery}}C} = -\frac{dt}{RC}
\]

Integrating both sides we get…….

\[
\ln(Q(t) - V_{\text{battery}}C) = -\frac{t}{RC}
\]

exponentiating both sides this becomes...

\[
Q(t) - V_{\text{battery}}C = Ae^{-\frac{t}{RC}}
\]

or....

\[
Q(t) = Ae^{-\frac{t}{RC}} + V_{\text{battery}}C
\]

where \(A\) is a constant to be determined by the conditions of the problem.. in this case if we are charging the capacitor from zero charge then \(Q(t) = 0\) so..

\[
Q(t) = V_{\text{battery}}C \left( 1 - e^{-\frac{t}{RC}} \right)
\]
The charge $Q(t)$ on the capacitor is given by:

$$Q(t) = V_{\text{battery}} C \left( 1 - e^{-\frac{t}{RC}} \right)$$
Current flow in the system..

\[ \frac{dQ}{dt} = V_{\text{battery}} C (e^{-\frac{t}{RC}}) \]

Note, that as the capacitor is charged the current gradually stops flowing around the circuit.
A question to ponder…

- Suppose $V_{\text{battery}} = 20\text{V}$, $R = 10\Omega$ and $C = 2.0\mu\text{F}$. Find $Q_f$, $I_0$ and the time constant of this circuit.
\[ Q(t = \infty) = V_{\text{battery}} \, C \left( 1 - e^{-\frac{\infty}{RC}} \right) = \]

\[ V_{\text{battery}} \, C = 20V \times (2 \times 10^{-6} F) = 40 \times 10^{-6} C \]

\[ I_0 = \frac{V_{\text{battery}}}{R} = 2.0 A \]

\[ RC = 10 \Omega \times (2 \times 10^{-6}) = 20 \times 10^{-6} \text{ seconds} \]
Now..discharging the capacitor

- Start with the capacitor charged and connect the terminals together through a resistor.
Using Kirchhoff’s Laws...

starting from...

\[- R \frac{dQ (t)}{dt} - \frac{Q (t)}{C} = 0\]

We can do some algebra to rewrite this as....

\[\frac{dQ (t)}{dt} = - \frac{Q (t)}{RC}\]

one last bit of algebra and then we can integrate. ... 

\[\frac{dQ (t)}{Q (t)} = - \frac{dt}{RC}\]
\[ \ln(Q(t)) = -\frac{t}{RC} \]

Exponentiating both sides this becomes...

\[ Q(t) = Ae^{-\frac{t}{RC}} \]

where \( A \) is a constant to be determined by the conditions of the problem.
In this case if we are discharging the capacitor from full charge then...

\[ Q(t) = V_{\text{battery}}C \left( e^{-\frac{t}{RC}} \right) \]
Household wiring
Exam 2 Recap Chapter 24

• The nature of capacitors and how to calculate a quantity that measures their ability to store charge.
• How to analyze capacitors connected in networks.
• How to calculate the energy stored in a capacitor.
• What is a dielectric and how they make capacitors more effective.
Recap Chapter 25

- The meaning of electric current, and how charges move in a conductor.
- What is meant by resistivity and conductivity of a substance.
- How to calculate resistance of a conductor from its dimensions and its resistivity.
- How an electromotive force (EMF) makes it possible for current to flow in a circuit.
- How to do calculations involving energy and power in circuits.
Recap Chapter 26

• How to analyze circuits with multiple resistors in series and in parallel.
• Rules that you can apply to analyzing circuits with more than one loop.
• How to use the ammeter, voltmeter, ohmmeter, or potentiometer in a circuit.
• How to analyze circuits that include both a resistor and capacitor.
• How electric power is distributed in the home.
From the formula sheet

Capacitance [F] (definition)
\[ C \equiv \frac{Q}{\Delta V} \]
(parallel-plate capacitance)
\[ C = \varepsilon_0 \frac{A}{d} \]

Electrostatic potential energy [J] stored in capacitance
\[ U_E(t) = \frac{1}{2} \frac{Q(t)^2}{C} \]

Electric dipole moment (2a = separation between two charges)
\[ |\vec{p}| = 2aq \]

Torque on electric dipole moment
\[ \vec{\tau} = \vec{p} \times \vec{E} \]

Potential energy of an electric dipole moment
\[ U = -\vec{p} \cdot \vec{E} \]

<table>
<thead>
<tr>
<th>Current [A]</th>
<th>(definition) with motion of charges</th>
<th>[ I = \frac{\Delta Q(t)}{\Delta t} ]</th>
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</thead>
<tbody>
<tr>
<td>[ I = nqv_{	ext{avg}} A ]</td>
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<tr>
<th>Current density [A/m²]</th>
<th>[ J = \frac{I}{A} ] (where ( I = \int \vec{J} \cdot \vec{n} , dA ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \rho = \frac{</td>
<td>J</td>
</tr>
<tr>
<td>[ R = \frac{V}{I} ]</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Resistivity [Ω·m]</th>
<th>(definition) for uniform cross-sectional area A</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \rho = \frac{L}{A} ]</td>
<td></td>
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<tr>
<td>[ P = \frac{I^2 R}{V^2/R} = IV ]</td>
<td></td>
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<tr>
<td>[ \tau_{\text{RC}} = \frac{RC}{V} ]</td>
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<thead>
<tr>
<th>Resistance [Ω]</th>
<th>[ q(t) = q_f (1 - e^{-t/\tau_{\text{RC}}}) ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ q(t) = q_0 e^{-t/\tau_{\text{RC}}} ]</td>
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