PROBLEM 1 (20 points)

You are given the circuit shown in the figure above. Each capacitor has a value of 4.0 \mu F and the voltage on the battery is 24.0 V.

(A) (5 pts) Find the charge on each of the capacitors.

\[ \text{Total capacitance} = \frac{12}{5} \mu F \text{ charged by} 24V \]
\[ Q_{\text{TOT}} = \left( \frac{12}{5} \right) \mu F \times 24V = 57.6 \mu C \]

Then \[ Q_{C_4} = 57.6 \mu C \]

(B) (5 pts) Find the potential difference across each capacitor.

\[ \text{See table below} \]

(C) (5 pts) Find the potential difference between points a and d in the circuit.

\[ V_d - V_a = V_{C_4} = -9.6V \]

(D) (5 pts) Find the energy stored in capacitor \( C_4 \).

\[ U = \frac{1}{2} Q_{C_4} V_{C_4} = 4.14 \times 10^{-4} \text{ Joules} \]

<table>
<thead>
<tr>
<th>( C_1 )</th>
<th>( C_2 )</th>
<th>( C_3 )</th>
<th>( C_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charge</td>
<td>( 19.2 \mu C )</td>
<td>( 19.2 \mu C )</td>
<td>57.6 \mu C</td>
</tr>
<tr>
<td>Voltage</td>
<td>4.8V</td>
<td>4.8V</td>
<td>9.6V</td>
</tr>
<tr>
<td>Energy</td>
<td>4.14 \times 10^{-4} \text{ Joules}</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
PROBLEM 2 (20 points)

You are given the circuit shown above.

(A) (5 pts) Find the current flowing in this circuit.

\[
16\text{V} - 12\text{V} = I \left(1.7\Omega + 5\Omega + 1.3\Omega + 10\Omega\right)
\]

\[
4\text{V} = I \left(10\Omega\right) \implies I = \frac{4\text{V}}{10\Omega} = 0.4\text{A}
\]

(B) (5 pts) Find the terminal voltages for each of the batteries when this current is flowing.

\[
V_a - V_b = \varepsilon_{uf} - IR = 16\text{V} - (0.4\text{A})(1.7\Omega) = 16\text{V} - 3.4\text{V} = 12.626\text{V}
\]

\[
V_c - V_d = \varepsilon_{uf} + IR = 12\text{V} + (0.4\text{A})(1.3\Omega) = 12\text{V} + 0.52\text{V} = 12.52\text{V}
\]

(C) (5 pts) What is the power being dissipated by the resistors external to the batteries when this current is flowing?

\[
\text{Power dissipated} = \sum I^2R = (0.4\text{A})^2 \left(5\Omega + 10\Omega\right)
\]

\[
= 0.1726\text{Watt}
\]

(D) (5 pts) Draw a sketch of the voltage as a function of position as you move around this loop starting at point b and making a complete trip returning to point b. Be sure to label your axes carefully.
PROBLEM 3 (20 points)

You are given the circuit shown above. Each of the resistors is a lamp with a resistance of 4.5Ω and the battery EMF is 9.0 V. Based on this information, answer the following:

(A) (5 pts) Find the current through each bulb. Make sure you label your answers.

\[ 9.0V = I (R + \frac{R}{3}) = \frac{4}{3}R \quad \Rightarrow \quad I = \frac{9}{4} \Omega = 1.5A \]

(B) (5 pts) Find the power being dissipated by each bulb. Again make sure that you label your answers clearly.

\[ P_{\text{total}} = I^2 R \quad \text{so for each bulb} \]

\[ P_{R_1} = I^2 R = 10.11 \text{ Watts} \]
\[ P_{R_2} = P_{R_3} = P_{R_4} = \frac{I^2}{3} R = 1.12 \text{ Watts} \]

(C) (5 pts) Which of these bulbs glows the brightest? Why?

Which glows the brightest? Brightness is proportional to the voltage rating so the more power dissipated, the brighter the bulb. Hence, \( R_1 \) is the brightest.

(D) (5 pts) If \( R_4 \) is removed, which bulb now glows the brightest?

When \( R_4 \) is removed the current in the circuit will change however even with only 2 bulbs remaining in parallel in the circuit, the power dissipated by \( R_1 \) is still highest.

\[ 9V = \left(\frac{3}{2}R\right) I \quad \Rightarrow \quad I = \frac{2(9)}{3R} = 1.33A \]

\[ P_{R_1} = (1.33)^2 \times 4.5\Omega = 8 \text{ Watts} \]
\[ P_{R_4} = \left(\frac{1.33}{2}\right)^2 \times 4.5\Omega = 2 \text{ Watts} \]
**PROBLEM 4 (20 points)**

Consider the multiloop circuit shown in the figure above. Answer the following using the variable assignments supplied in the figure.

(A) (4 pts) Based on the assignments of currents above, what is the algebraic relationship between currents $i_1$, $i_2$ and $i_3$?

$$i_1 + i_2 = i_3$$

(B) (4 pts) Using Kirchoff's Laws, write down the "loop" equation for Loop 1 shown in the figure.

$$5V - 10V = i_2(1Ω + 4Ω) - i_1(2Ω + 3Ω)$$

$$-5V = 5i_2 - 5i_1$$

(C) (4 pts) Using Kirchoff's Laws, write down the "loop" equation for Loop 2 shown in the figure.

$$5V = i_2(1Ω + 4Ω) + i_3(10Ω)$$

$$5V = 5i_2 + 10i_3$$

(D) (4 pts) Using Kirchoff's Laws, write down the "loop" equation for Loop 3 shown in the figure.

$$10V = i_1(2Ω + 3Ω) + i_3(10Ω)$$

$$10V = 5i_1 + 10i_3$$

(C) (4 pts) What are the values of $i_1$, $i_2$ and $i_3$ for this circuit?

Loop 1:

$$-5 = -5i_1 + 5i_2$$

Loop 2:

$$5 = 5i_2 + 10i_3$$

$$0 = 5i_1 + 20i_2$$

$$i_1 = -4i_2$$

$$i_2 = -2A$$

$$i_3 = 0.8A - 2A = -1.2A$$

Loop 3:

$$-5 = -5i_1 + 5i_2 + 20i_2 + 5i_3$$

$$25i_2 = -5i_1$$

$$i_1 = +0.8A$$
PROBLEM 5 (20 points)

Consider the circuit shown above. Initially the switch is in position 1 and the capacitor is uncharged. At t=0 the switch is moved to position 2 and you are to consider the circuit from that time forward. The capacitor has a value of 6.0μF and the EMF has a voltage of 30.0 V.

(A) (5 pts) What will be the charge on the capacitor after waiting a long time after the switch has been moved to position 2?

\[ Q_{\text{fully charged}} = CV = (6.0 \mu F)(30.0V) = 180 \mu C \]

(B) (5 pts) How much current will be flowing in the circuit when that charge has been reached?

When capacitor is charged, current stops flowing in the circuit. \[ I = 0 \]

(C) (5 pts) If after the switch is in position 2 for 5.0 msec the charge on the capacitor is 100.0μC. What is the value of the resistor in the circuit?

\[ Q(t = 5 \times 10^{-3} \text{sec}) = q_f (1 - e^{-t/RC}) = 100.0 \mu C \]

\[ \frac{180 \mu C}{100.0 \mu C} (1 - e^{-t/RC}) = 1 \]

\[ (1 - e^{-5 \times 10^{-3}/RC}) = \frac{8}{9} \]

\[ e^{-5 \times 10^{-3}/RC} = \frac{1}{9} \]

\[ -5 \times 10^{-3}/RC = \ln (\frac{1}{9}) \]

\[ t = -RC \ln (0.1) = 2.8 \text{ms} \]

(D) (5 pts) How long after the switch is in position 2, will it take for the charge on the capacitor to reach 99.0% of the final value of the charge?

\[ RC = \frac{5 \times 10^{-3}}{0.0061} = 818 \]

\[ R = \frac{0.0061}{C} = 1027 \Omega \]