Lecture 2: Units and Vectors

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Chapter 1: Math n’ Stuff

• Won’t cover the entire chapter:
  – Unit conversions
  – Problem Solving
    • Tricks
    • Methods
  – Vectors
    • Components (Unit vectors)
    • Addition
    • Multiplication (dot and cross products)
Clickers Setup

• Turn on your clicker (press the power button)

• Set the frequency:
  – Press and hold the power button
  – Two letters will be flashing
  – If it’s not “BA”, press “B” and then “A”

• If everything works, you should see “Welcome” and “Ready”
Clickers Setup

• Turn on your clicker (press the power button)
• Set the frequency:
  – Press and hold the power button
  – Two letters will be flashing
  – If it’s not “BD”, press “B” and then “D”
• If everything works, you should see “Welcome” and “Ready”
Clicker Question 1

• Do you have your i>clicker with you today?

   A) Yes
   B) No
   C) Maybe
   D) I like pudding

We use the “BD” frequency in this class
Clicker Question 3

- Who do you think will be elected in 2016 as the next US President?

A) Hillary Clinton
B) Karl Marx
C) Abraham Lincoln
D) Britney Spears

We use the “BD” frequency in this class
Derivatives

• What is the derivative of $4x^2$?
  – 4x
  – 8x
  – 8

• What is the derivative of 100?
  – 100x
  – x
  – 0
Converting Units

• Problem: express length of a football field in feet:
  – 1 football field = 100 yards
  – 1 yard = 3 feet

• Solution:
  – 1 football field = 1 football field
  – 1 football field = 1 football field x (1) x (1)
    • Can always multiply by a unity; nothing should change
  – 1 football field = 1 football field x (100 yards/1 football field) = 100 yards
    • Got to yards above, which is already an improvement
  – 1 football field = 100 yards x (3 feet/yard) = 300 feet
    • Got rid of yards and expressed in feet!
  – Both are units of length!

• Another problem:
  – Express speed \( v = 50 \text{ km/hour} \) in \( \text{m/sec} \)
Pre-Lecture Question

• You look it up and find that there are 2.54 centimetres in one inch

• The motorcycle engine on a Kawasaki Ninja 1000 has a displacement of 1043 cubic-centimeters (cm³). In order to calculate its engine displacement in cubic-inches (in³) what unit conversion factor would you use to multiply the given displacement?

A. 1 in³ / 2.54 cm³
B. 2.54 cm³ / 1 in³
C. 1 in³ / 16.4 cm³
D. 16.4 cm³ / 1 in³

Dimensional Analysis: Question 1 (N = 99)
Problem Solving Overview

- There are good general problem solving TRICKS
  - Units checking
  - Special case checking
  - Etc.

- There are good METHODS of problem solving that prepare you for the exams

We’ll use both to solve problems in lecture
First Things First!

What’s the first thing you should do when you’re given a problem?

• **Draw a diagram!!!** *Trick #1*
  – Usually good for some partial credit

• List givens and wants as variables
  – Also a good bet for partial credit

Then use reasonable equations and solve with variables
Trick #2: Units

• The speed of your car isn’t measured in seconds, it’s measured in meters/second (or miles/hour etc.)

• Paying attention to the units will help you catch LOTS of mistakes on exams, quizzes and homework!!

  – If we ask what the mass of your car is, make sure your answer is in kg (or lbs etc.)

**Trick #2: Every time you finish a problem **

**ALWAYS check the units of your answer!!**
Tricks #3 and #4

Check *Reasonableness*:

- Can you find another way to do the same problem that gives the same answer? *Trick #3*

- Trivial choices of values for variables give expected numerical answers? Example: Zero, or infinity *Trick #4*
Moving toward an Example Problem

• Next we’ll do an example problem like one of the homework problems in the text book

• Solve this problem using the right method
  – Draw a diagram
  – Convert the numbers to variables
  – Solve to get a formula
  – Plug in the numbers at the end
  – Check
Example Problem

You want to measure the height of a building. You stand 2m away from a 3m pole and see that it’s “in line” with the top of the building. You measure 16 m from the pole to the building.

What is the height of the building?
Vectors

Vectors:

–Why we care about them
–Addition & Subtraction
–Unit Vectors
–Multiplication
Why do we care about Vectors?

• As you may have noticed, the world is not one-dimensional
• Three dimensions: X, Y and Z. Example:
  1. Up from us
  2. Straight in front of us
  3. To the side from us
     – All at 90 degrees from each other. Three dimensional axis.
• Need a way of saying how much in each direction

For this we use VECTORS
Vector and Scalar

- Vectors have a magnitude **AND** a direction
  - 10 miles in the south direction
- Scalars are just a number
  - Mass of your car
  - Earth radius
Where am I?

- Let’s say I’m here
- You’re here (origin)
- I call you on the cell phone.
- How do I tell you how to get to me?
- 2 equivalent ways:
  - Travel 11.2 km at an angle of 26.5 degrees
  - Travel 10 km East then 5 km North

A single vector in arbitrary direction can be thought of as two vectors in nice simple directions (like $X$ and $Y$). This can make things much easier.
Vector Addition

- To specify where I am, often doing the two vector version is easier

Represent Graphically:
- Lay down first vector
- Lay down second vector
  - Put the tail at the head of the first vector
- The “Sum” is where I am
Re-write my location

- Describe my location in terms of the sum of two vectors
  \[
  \vec{R} = \vec{R}_X + \vec{R}_Y
  \]
  \[
  |R_X| = |R| \cos \Theta
  \]
  \[
  |R_Y| = |R| \sin \Theta
  \]
- Careful when using the sin and cos
Specifying a Vector

• Two equivalent ways:
  – Components $V_x$ and $V_y$
  – Magnitude $V$ and angle $\theta$

• Switch back and forth
  – Magnitude of $V$
    $$|V| = (v_x^2 + v_y^2)^{\frac{1}{2}}$$
    • Pythagorean Theorem
  – $\tan \theta = \frac{v_y}{v_x}$

• Either method is fine, pick one that is easiest for you, but be able to use both

\[
\begin{align*}
\sin \theta &= \frac{V_y}{V} \\
\cos \theta &= \frac{V_x}{V} \\
\tan \theta &= \frac{V_y}{V_x} \\
V^2 &= V_x^2 + V_y^2
\end{align*}
\]
Unit Vectors

Another notation for vectors:
- Unit Vectors denoted $\mathbf{i}, \mathbf{j}, \mathbf{k}$

$\hat{i}$ means 1 in the $x$ direction

$\hat{j}$ means 1 in the $y$ direction

$\hat{k}$ means 1 in the $z$ direction

$$\mathbf{V} = V_x \hat{i} + V_y \hat{j} + V_z \hat{k}$$
Unit Vectors

Similar notations, but with \( x, y, z \)

\( \hat{x} \) is the same as \( \hat{i} \)
\( \hat{y} \) is the same as \( \hat{j} \)
\( \hat{z} \) is the same as \( \hat{k} \)

\[
\vec{V} = V_x \hat{x} + V_y \hat{y} + V_z \hat{z}
\]
Vector in Unit Vector Notation

\[ |V_x| = |V| \cos \theta \]
\[ |V_y| = |V| \sin \theta \]
\[ \vec{V} = \vec{V}_x + \vec{V}_y \]
\[ \vec{V} = V_x \hat{i} + V_y \hat{j} \]
\[ \vec{V} = |V| \cos \theta \hat{i} + |V| \sin \theta \hat{j} \]
General Addition Example

- Add two vectors using the $i$-hats, $j$-hats and $k$-hats.

$$\vec{D}_R = \vec{D}_1 + \vec{D}_2$$

$$\vec{D}_1 = 10 \text{ km } \hat{i} + 0 \text{ km } \hat{j} + 0 \text{ km } \hat{k}$$

$$\vec{D}_2 = 0 \text{ km } \hat{i} + 5 \text{ km } \hat{j} + 0 \text{ km } \hat{k}$$

$$\Rightarrow \vec{D}_R = 10 \text{ km } \hat{i} + 5 \text{ km } \hat{j} + 0 \text{ km } \hat{k}$$
Simple Multiplication

• Multiplication of a vector by a scalar
  – Let’s say I travel 1 km east. What if I had gone 4 times as far in the same direction?
  → Just stretch it out, multiply the magnitudes

• Negatives:
  – Multiplying by a negative number turns the vector around
Subtraction

Subtraction is easy:

- It’s the same as addition but turning around one of the vectors. I.e., making a negative vector is the equivalent of making the head the tail and vice versa. Then add:

\[ \vec{V}_2 - \vec{V}_1 = \vec{V}_2 + (-\vec{V}_1) \]
Vector Question

• Vector A has a magnitude of 3.00 and is directed parallel to the negative y-axis and vector B has a magnitude of 3.00 and is directed parallel to the positive y-axis. Determine the magnitude and direction angle (as measured counterclockwise from the positive x-axis) of vector C, if C=A−B.

A. C=0.00 (its direction is undefined)
B. C = 3.00; θ = 270°
C. C = 3.00; θ = 90°
D. C = 6.00; θ = 270°
F. C = 6.00; θ = 90°
Vector Question

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E. C = 6.00; θ = 90°
How do we Multiply Vectors?

- First way: **Scalar Product or Dot Product**
  - Why Scalar Product?
    - Because the result is a scalar (just a number)
  - Why a Dot Product?
    - Because we use the notation $A \cdot B$

$$A \cdot B = |A||B|\cos \theta$$
First Question:

\[ \vec{A} \cdot \vec{B} = |A||B|\cos \Theta \]

What is \( \hat{i} \cdot \hat{j} \)?
First Question:

What is \( \hat{i} \bullet \hat{j} \)?

A) 1  
B) 0  
C) -1  
D) unit vector \( \hat{k} \)
Harder Example

$$\vec{A} = A_X \hat{i} + A_Y \hat{j}$$

$$\vec{B} = B_X \hat{i} + B_Y \hat{j}$$

What is $\vec{A} \cdot \vec{B}$ using Unit Vector notation?
Vector Scalar Product

- Calculate the scalar product $\mathbf{A} \cdot \mathbf{B}$:

A. 11.6  
B. 12.0  
C. 14.9  
D. 15.4  
E. 19.5  

$|\mathbf{A}| = 6.0$

$\mathbf{B} = 3.2\mathbf{i} - 0.6\mathbf{j}$
Vector Scalar Product

- Calculate the scalar product $A \cdot B$:

A. 11.6  
B. 12.0  
C. 14.9  
D. 15.4  
E. 19.5
Vector Cross Product

\[ \vec{C} = \vec{A} \times \vec{B} \]

\[ |C| = |A| |B| \sin \Theta \]

• This is the last way of multiplying vectors we will see
• Direction from the “right-hand rule”
• Swing from A into B!
Vector Cross Product Cont...

- Multiply out, but use the \( \sin \theta \) to give the magnitude, and RHR to give the direction

\[
\vec{C} = \vec{A} \times \vec{B}
\]

\[
|C| = |A| |B| \sin \Theta
\]

\[
\hat{i} \times \hat{i} = 0 \quad (\sin \theta = 0)
\]

\[
\hat{i} \times \hat{j} = \hat{k} \quad (\sin \theta = 1)
\]

\[
\hat{i} \times \hat{k} = -\hat{j} \quad (\sin \theta = 1)
\]
Cross Product Example

\[ \vec{A} = A_X \hat{i} + A_Y \hat{j} \]
\[ \vec{B} = B_X \hat{i} + B_Y \hat{j} \]

What is \( \vec{A} \times \vec{B} \) using Unit Vector notation?
Vector Scalar Product

- Calculate the vector product $\mathbf{C} = \mathbf{A} \times \mathbf{B}$:
- Vector $\mathbf{A}$ points in positive y direction and has magnitude of 3
- Vector $\mathbf{B}$ points in negative x direction and has magnitude of 3

- Which is the correct way to calculate $\mathbf{C}$?

A. $\mathbf{C} = 3\mathbf{j} \times (-3\mathbf{i}) = -9\mathbf{k}$
B. $\mathbf{C} = 3\mathbf{j} \times (-3\mathbf{i}) = +9\mathbf{k}$
C. $\mathbf{C} = 3 \times (-3) \times \sin (90^\circ) = -9$
D. $\mathbf{C} = 3 \times (-3) \times \sin (270^\circ) = +9$
For Next Time

- Homework (via masteringphysics.com)
  - Math and stuff: try to finish ASAP
    • Nominal deadline is Sunday of the coming week, but don’t wait

- Pre-lectures for next week: due Sunday!
  - Read Chapter 2, prepare for in-lecture quizzes
  - Start working on homework problems for Ch. 2

- In Lecture: Chapter 2, quizzes
- Recitation: Chapter 1
  • Do everything you can in the HW before the recitation!

- Lab (WebAssign!):
  - Prepare for the lab, complete all read pre-lab parts before the lab!