Final Exam Information

Final Exam: Friday Dec. 12
Place: MPHY 205
Time: 7:30-9:30 AM

Exam Structure:
5 multiple choice
4 “work-out” problems

Chapters covered on the exam:

Yes you still have to do the homework for chapters 14 and 15
Problem 2

A solid disk plate with diameter of 0.200m and mass 0.400 kg is rotating with an angular speed of 5.00 rad/s. A non-rotating rod with length \( L = 0.150 \text{m} \) is dropped onto the plate. Now the system is observed to rotate at 3 rad/s. If the rod and disk rotate about a common axis through their centers, what is the moment of inertia of the rod? What is the mass of the rod?

\[
\begin{align*}
I_{\text{rod}} &= \frac{1}{12} mL^2 \\
I_{\text{disk}} &= \frac{1}{2} MR^2 \\
\omega_1 &= 5.00 \text{ rad/s} \\
\omega_2 &= 3.00 \text{ rad/s}
\end{align*}
\]

\[
L_i = I \omega = \frac{1}{2} MR^2 \omega_1
\]

\[
\frac{1}{2} MR^2 \omega_1 = \left( \frac{1}{2} MR^2 + I_{\text{rod}} \right) \omega_2
\]

\[
I_{\text{rod}} = \frac{1}{2} ML^2 (\omega_1 - \omega_2)
\]

\[
I_{\text{rod}} = \frac{1}{2} ML^2 \\
M = \frac{12 I_{\text{rod}}}{L^2} = \frac{12 (0.0033 \text{ kg m}^2)}{(1.50 \text{ m})^2} = 0.709 \text{ kg}
\]
Problem 4

A pulley with radius \( R = 18.0 \text{ cm} \) has a moment of inertia of \( I = 0.29 \text{ kg m}^2 \). A nonslip rope over this pulley has a mass hanging on each end. On the left side \( m_1 = 1.40 \text{ kg} \) and on the right side \( m_2 = 8.90 \text{ kg} \).

a) Sketch the system and draw free body diagrams for each mass and the pulley.

b) When the system is released from rest what is the acceleration of \( m_2 \)?

c) What is the tension in each side of the rope?

\[ \Sigma F_y = T_1 - m_1 g = m_1 a \]

\[ \Sigma F_y = T_2 - m_2 g = -m_2 a \]

\[ \Sigma \tau = T_1 R - T_2 R = -I \alpha \]

\[ (T_1 - T_2) R = -I \alpha \]

\[ (T_1 - T_2) = \frac{m_1 a + m_1 g - (m_2 g - m_2 a)}{R^2} \]

Solve for \( a \):

\[ m_1 a + m_2 a + I \alpha = m_2 g - m_1 g \]

\[ a = \frac{(m_2 - m_1) g}{m_1 + m_2 + I/R^2} = \frac{3.82 \text{ m/s}^2}{5 \text{ kg}} \]
Problem 4 – cont.

b) When the system is released from rest what is the acceleration of \( m_2 \)?

c) What is the tension in each side of the rope?

\[
\begin{align*}
T_1 - m_1g &= m_1a \\
T_2 - m_2g &= -m_2a
\end{align*}
\]

\[
T_1 = m_1(a+g) = 1.40\text{kg}(3.82\text{m/s}^2 + 9.8\text{m/s}^2) = \boxed{9.1N}
\]

\[
T_2 = m_2(g-a) = 8.90\text{kg}(9.8\text{m/s}^2 - 3.82\text{m/s}^2) = \boxed{53.2N}
\]
A bullet, $m_1 = 0.04$ kg, with a velocity of 285 m/s hits a block, $m_2 = 2.3$ kg, that is initially at rest. The bullet passes through the block and emerges with a velocity of 85 m/s.

a) What is the velocity of the block after the bullet emerges?

b) What is the total kinetic energy of the system before and after the collision?

\[ V_b = 285 \text{ m/s}, \quad V_{bi} = 85 \text{ m/s} \]

\[ V_{bf} = \frac{m_b (V_{bi} - V_{bf})}{m_b} = \frac{0.04 \text{ kg} \times (285 \text{ m/s} - 85 \text{ m/s})}{2.3 \text{ kg}} = 3.48 \text{ m/s} \]

\[ K_i = \frac{1}{2} m_b V_{bi}^2 = 1624 \text{ J} \]

\[ K_f = \frac{1}{2} m_b V_{bi}^2 + \frac{1}{2} m_b V_{bf}^2 = 158 \text{ J} \]
Problem 9

A uniform meter stick of mass $1.5\text{kg}$ is attached to the wall by a frictionless hinge at one end. On the opposite end it is supported by a vertical massless string such that the stick makes an angle of $40^\circ$ with the horizontal.

Find the tension in the string and the magnitude and direction of the force exerted on the stick by the hinge. Suppose the string is cut. Find the angular acceleration of the stick immediately thereafter.
Problem 9

\[ \sum F = \frac{1}{2} \theta mg \cos \theta = -I \alpha \]

\[ I_{rots, ext} = \frac{1}{3} mL^2 \]

\[ \frac{1}{2} mg \cos \theta = \frac{1}{3} mL^2 \alpha \]

\[ \alpha = \frac{3 \ g \cos \theta}{2L} = \frac{3}{2} \left( \frac{9.8 \text{ m/s}^2}{1 \text{ m}} \right) \cos 40^\circ \]

\[ \alpha = 11.27 \text{ rad/s} \]

\[ \square C - w \]
Problem 10

The Earth has an orbital radius around the sun of \(1.50 \times 10^8\) km; Mars has an orbital radius of \(2.28 \times 10^8\) km.

In order to send a spacecraft from the Earth to Mars, it is convenient to launch the spacecraft into an elliptical orbit whose perihelion coincides with the orbit of Earth and whose aphelion coincides with the orbit of Mars.

a) What is the semi-major distance of such an orbit? What is its eccentricity?

b) To achieve such an orbit what speed relative to the Earth must the spacecraft be launched? (Ignore the pull of the gravity from the Earth and Mars on the spacecraft)

\[
\begin{align*}
2a &= R_m + R_E \\
a &= \frac{1.50 \times 10^8\text{m} + 2.28 \times 10^8\text{m}}{2} \\
a &= 1.89 \times 10^8\text{m} \\
R_E &= a - ea \\
e &= 1 - \frac{R_E}{a} = 1 - \frac{1.50 \times 10^8\text{m}}{1.89 \times 10^8\text{m}} = 0.206
\end{align*}
\]
b) To achieve such an orbit what speed relative to the Earth must the spacecraft be launched? (Ignore the pull of the gravity from the Earth and Mars on the spacecraft)

\[ \vec{L} = \vec{r} \times \vec{p} \]

\[ R_E M V_E = R_m M V_m \]

Need orbital velocity \( \beta \) of Mars

\[ F = ma = \frac{MV_m^2}{R_m} = \frac{GM_0}{R_m} \]

\[ V_m = \sqrt{\frac{GM_0}{R_m}} = \sqrt{\frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30}}{2.28 \times 10^9}} \]

\[ V_m = 2.41 \times 10^4 \text{ m/s} \]

\[ V_E = \frac{R_m V_m}{R_E} = \frac{(2.28 \times 10^9)(2.41 \times 10^4)}{1.50 \times 10^8} \]

\[ V_E = 3.67 \times 10^4 \text{ m/s} \]
Problem 11

You are to design a roller coaster in which cars start from rest a height $h = 30$ m then rolls down into a valley, then up a mountain.

a) What is the speed of the cars at the bottom of the valley?

b) If the passengers feel 8g at the bottom of the valley what must be the radius $R$ of the arc of the circle that fits in the bottom of the valley?

c) The top of the mountain is an arc of the same radius $R$. If the passengers are to feel 0g at the top of the mountain what must be its height $H$?

\[ \begin{align*}
\text{a) } & v_1 = mg - h \quad K_1 = 0 \\
v_2 = 0 \quad K_2 = \frac{1}{2}m v_z^2 \\
v_z = \sqrt{2gh} = 12.4 \text{ m/s} \\
\text{b) } & F_y = N - mg = ma_r \quad m(8g) - mg = m\frac{v_z^2}{R} \\
& R = \frac{v_z^2}{7g} = \frac{(24.3 \text{ m/s})^2}{7(9.8 \text{ m/s})^2} = 8.61 \text{ m} \\
\end{align*} \]
Problem 11 cont.

(c) solving for $H$ if $0 \leq \theta < \pi$

\[
\sum F_y = 0 - mg = -\frac{mv_3^2}{R}
\]

\[
V_3 = \sqrt{R_0} = 9.19\, \text{m/s}
\]

\[
\frac{1}{2} m v_2^2 = \frac{1}{2} m v_3^2 + mgH
\]

\[
H = \frac{v_2^2 - v_3^2}{2g} = \frac{(24.3\, \text{m/s})^2 - (9.19\, \text{m/s})^2}{2(9.8\, \text{m/s})}
\]

\[
H = 25.8\, \text{m}
\]
Problem 8

A solid disk with radius $R = 0.35 \text{m}$ and a mass $m = 4.2 \text{ kg}$ rolls down a curved ramp without slipping. It starts at rest and drops a vertical distance of $14.7 \text{ m}$. It leaves the ramp traveling horizontally, it falls another $3.7 \text{ m}$ before striking the ground.

a) At the bottom of the ramp what is its kinetic energy?

b) How far from the end of the ramp does it land?
A projectile launched vertically upward from the surface of the moon rises to a maximum altitude of 865 km. What was the projectile's initial speed? \( M_{\text{moon}} = 7.35 \times 10^{22} \) kg \( R_{\text{moon}} = 1.74 \times 10^6 \) m

\[
\begin{align*}
U_1 &= -\frac{G M_{\text{moon}}}{R_{\text{moon}}} \\
U_2 &= -\frac{G M_{\text{moon}}}{(R_{\text{moon}} + h)} \\
K_1 &= \frac{1}{2} m v^2 \\
K_2 &= 0
\end{align*}
\]

\[
\frac{1}{2} m v^2 - \frac{G M_{\text{moon}}}{R_{\text{moon}}} = -\frac{G M_{\text{moon}}}{(R_{\text{moon}} + h)}
\]

\[
v^2 = 2 G M_{\text{moon}} \left( \frac{1}{R_{\text{moon}}} - \frac{1}{(R_{\text{moon}} + h)} \right)
\]

\[
v^2 = 2 \left( 6.67 \times 10^{-11} \right) \left( 7.35 \times 10^{22} \right) \left( \frac{1}{1.74 \times 10^6} - \frac{1}{(1.74 \times 10^6 + 8.65 \times 10^5)} \right)
\]

\[
v^2 = 1.87 \times 10^6 \text{ m/s}^2
\]

\[
v = 1.37 \times 10^3 \text{ m/s}
\]