• Your scores will be posted after class.

Score Range = Approx. letter Grade
66 – 100 = A
50 – 65 = B
30 – 49 = C
15 – 29 = D
0 – 14 = F

(You can pickup your exam after class today or on Wednesday)

Note: I will only be in my office from 2:30 – 3:00 PM tomorrow
Wednesday

- Rotational motion review
- Equations of motion for rotation
- Kinetic energy of rotation
- Moment of inertia
- Conservation of energy with rotation

Today

- Ch 9 examples
- Rotation of a rigid body
- Torque
- Angular momentum
- Conservation of angular momentum
- Work and power with torque
Example Calculating moment of inertia

3 small blocks, each with different masses, are clamped at the ends and at the center of a rod of length $L$ and negligible mass. Compute the moment of inertia of the system about an axis perpendicular to the rod and passing through (a) the center of the rod and (b) a point one-fourth of the length from the end.

(a) @ $L/2$

$$I = \sum_i m_i r_i^2$$

$$= (2M)(\frac{L}{2})^2 + 1.5M(0)^2 + M(\frac{L}{2})^2$$

$$= \frac{3}{4}ML^2 = .75ML^2$$

(b) @ $L/4$

$$I = (2M)(\frac{L}{4})^2 + 1.5M(\frac{L}{4})^2 + M(\frac{3L}{4})^2$$

$$= .781ML^2$$
Example: Moment of Inertia and Energy

The pulley in the figure has a radius $R$ and a moment of inertia $I$. The rope does not slip over the pulley and the pulley spins on a frictionless axle. The coefficient of kinetic friction between block A and the tabletop is $\mu_k$.

The system is released from rest, and the block B descends. Block A has a mass $m_A$ and block B a mass of $m_B$. Use energy methods to calculate the speed of block B as a function of the distance $d$ that it has descended.

\[ V_{\text{block}} = R \omega_{\text{pulley}} \]

\[ \text{Cons. Eq. Energy} \]
\[ K_i + U_i + W_{\text{kin}} = K_f + U_f \]
\[ K_i = 0 \]
\[ U_i = m_Bgd \]
\[ U_f = 0 \]
\[ m_Bgd - \mu_k m_Agd = \frac{1}{2} m_A v^2 + \frac{1}{2} m_B v^2 + \frac{1}{2} I \omega^2 \]

\[ \frac{v^2}{2} = \frac{2(m_Bgd - \mu_k m_Agd)}{(m_A + m_B + I/R^2)} \]
Prelecture question 1

A uniform solid sphere has a mass $M$ and radius $R$. The moment of inertia about an axis through its center is $(2/5)MR^2$.

What is the moment of inertia of the sphere about a parallel axis that is tangent to the surface of the sphere?

\[ I_{\text{surface}} = (2/5)MR^2 \]

\[ I_{\text{surface}} = MR^2 \]

\[ I_{\text{surface}} = (7/5)MR^2 \]

\[ I = I_{CM} + I_p \]
\[ = \frac{2}{5}MR^2 + MR^2 \]

First Answer Choice Distribution (N = 71)
A thin, very light wire is wrapped around a drum that is free to rotate. The free end of the wire is attached to a ball of mass $m$. The drum has the same mass $m$. Its radius is $R$ and its moment of inertia is $I = (1/2)mR^2$. As the ball falls, the drum spins.

At an instant that the ball has translational kinetic energy $K$, the drum has rotational kinetic energy

- A. $K$
- B. $2K$
- C. $K/2$
- D. none of these

$$K_{\text{ball}} = 2K_{\text{Drum}}$$
• New physical quantity: Torque

• Torque is the rotational analog of force, it is what causes angular acceleration

We want to:

• analyze the motion of a body that has rotational and translational motion

• use work and power to solve problems for rotating bodies

• study angular momentum and how it changes (or doesn’t) with time
Cross Product: Again (*in case you forgot*)

Vector multiplication used in finding the torque and angular momentum:

The vector or cross product

- Results in a **vector** with a **magnitude** of

\(|\vec{C}| = |\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}| \sin \theta = AB \sin \theta|

- **Direction** is perpendicular to both vectors
  (Find the direction with the Right-Hand-Rule)

The cross product anti-commutes

\[ \hat{A} \times \hat{B} = -\hat{B} \times \hat{A} \]

You can write it out in terms of components also (via the determinant)

\[ \hat{A} \times \hat{B} = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k} \]
Torque

- What does it take to cause angular acceleration? A force that gives a twisting or turning action: Torque (τ)
- Torque is the quantitative measure of a force’s ability to change rotational motion about an axis (the rotational analogue of force)
- It depends on the linear force applied and the distance from the axis of rotation

\[ \vec{\tau} = \vec{r} \times \vec{F} \]

\[ \vec{\tau} = (F \sin \phi) r = F_{\text{tan}} r = F_{\perp} r \]

- \( \vec{F} \) is the applied force, and \( \vec{r} \) is the displacement from the axis of rotation
- If the axis of rotation is not defined then the torque can not be defined
- units are N*m (N*m=J but torque is not an energy)

Do not memorize "\((F \sin \phi) r\)"

\[ \tau = |\vec{F}| \left| \vec{r} \right| \sin \phi = F_{\perp} r \]

Direction from R.H.R

Counter-clockwise = positive rotation
Clockwise = negative rotation
Torque and angular acceleration

- The tangential component of a force may cause a rotation, which will change the speed of rotation, so it corresponds to a tangential acceleration

\[ F_{\text{tan}} = F_{\perp} = ma_{\text{tan}} \]

- If we put this in terms of the angular acceleration, (which we defined in Ch 9)

\[ a_{\text{tan}} / R = \alpha \rightarrow F_{\perp} = m(R\alpha) \]

\[ \tau = F_{\perp} R = mR(\alpha R) \rightarrow \tau = (mR^2)\alpha \]

\[ mR^2 = I \]

\[ \vec{\tau} = I\vec{\alpha} \]

- If we have many particles then

\[ \tau_A + \tau_B + ... = (m_A R_A^2 + m_B R_B^2 + ...)\alpha \]

\[ \sum \vec{\tau} = I_{\text{tot}} \vec{\alpha} \]

- Which is the rotational analog to

\[ \sum \vec{F} = m\vec{a} \]
The figure shows a force being applied to a pipe wrench of length L. What is the torque being applied?

a) $FL$

b) $FL \cos \theta$

c) $FL \sin \theta$

d) $FL \tan \theta$

e) 0

Remember:

$\vec{\tau} = \vec{r} \times \vec{F}$

$|\vec{\tau}| = F_{\perp} r$
Problem solving strategy

1) Draw a picture.

2) Free-body diagram for each body, including weight at the CoM

3) Identify the distances to axis of rotation and the positive/negative sense of rotation (if possible).

4) Find $I_{tot}$

5) Write out $\sum \vec{F} = m\vec{a}$ and/or $\sum \vec{\tau} = I_{tot}\vec{\alpha}$ and solve.
**Simple example: Teeter-totter**

A pile of angry birds (10kg) and a box of kittens (15kg) are playing on the teeter-totter. What is the net torque on the teeter-totter board (length \( L = 4.0\) m and mass \( M_t = 150\) kg) and the linear acceleration of each?

Remember:

\[
|\tau| = F_r r
\]

\[
\sum \tau = I \alpha
\]

\[
a_{\text{tan}} = \alpha R
\]

(b)

\[
a_{\text{tan}} = \alpha R
\]

\[
|a_{\text{tan}}| = \left(\frac{32\pi^2}{9}\right)(2m)
\]

\[
= 1.64 \text{ m/s}^2
\]

\[
FBD
\]

\[
\sum F = \tau
\]

\[
\sum F = \sum I = I_{\text{tot}} \alpha = \sum F_{\text{r}}
\]

\[
= W_B \left(\frac{L}{2}\right) - W_K \left(\frac{L}{2}\right)
\]

\[
= 10\text{kg}(9.8 \text{ m/s}^2)\left(\frac{4}{2}\right) - 15\text{kg}(4\text{m})^2\left(9.8\right)
\]

\[
= -98\text{N} \cdot \text{m}
\]

\[
-98\text{N} \cdot \text{m} = I_{\text{tot}} \alpha
\]

\[
-98\text{N} \cdot \text{m} = \frac{300\text{kg} \cdot \text{m}^2}{300\text{kg} \cdot \text{m}^2}
\]

\[
\alpha = -\frac{32\pi^2}{5}\text{ rad/s}^2
\]
In which of the cases shown is the torque provided by the applied force about the rotation axis biggest? In both cases the magnitude and direction of the applied force is the same.

a) case 1  
b) case 2  
c) they are the same
A ball of mass 3M at x=0 is connected to a ball of mass M at x=L by a massless rod. Consider the three rotation axes A, B and C as shown, all parallel to the y axis.

For which rotation axis is the moment of inertia of the object smallest? (It may help you to figure out where the center of mass of the object is.)

\[ X_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{3M \times 0 + M \times L}{4M} = \frac{L}{4} \]

\[ I_{\text{point}} = \sum m_i r_i^2 \]

\[ I_A = (3M)0^2 + ML^2 = ML^2 \]

\[ I_B = 3M \left( \frac{L}{4} \right)^2 + M \left( \frac{3L}{4} \right)^2 = .75ML^2 \]

\[ I_C = 3M \left( \frac{L}{2} \right)^2 + M \left( \frac{L}{2} \right)^2 = ML^2 \]
In Case 1, a force $F$ is pushing perpendicular on an object a distance $L/2$ from the rotation axis. In Case 2 the same force is pushing at an angle of 30 degrees a distance $L$ from the axis.

In which case is the torque due to the force about the rotation axis biggest?

- a) case 1
- b) case 2
- c) same

**Solution:**

1. For Case 1:
   
   $\vec{r} = \vec{r} \times \vec{F} = rF_\perp$

   $T_1 = \left( \frac{L}{2} \right) F$

2. For Case 2:
   
   $T_2 = LF \sin(30^\circ) = \frac{LF}{2}$

**Bar Chart:**

- A: 19.4
- B: 22.3
- C: 58.3

**Note:** The bar chart shows the distribution of students' answers.
Two wheels can rotate freely about fixed axles through their centers. The wheels have the same mass, but one has twice the radius of the other. Forces $F_1$ and $F_2$ are applied as shown such that the angular acceleration of the two wheels is the same.

Compare the magnitudes of the two forces

a) $F_2 = F_1$

b) $F_2 = 2F_1$

c) $F_2 = 4F_1$

\[
F_{\perp} = ma_{\tan}
\]

\[
= m(\alpha R)
\]
Example Rotational and translational forces

Remember our falling block unwinding a cable from Ch 9. What is the acceleration of the falling block and the tension in the cable?

\[ a_{\text{tan}} = R \alpha \]

\[ \sum F_y = T - mg = -ma \]

\[ \sum \tau = I \alpha = \frac{1}{2} MR^2 \frac{a}{R} \]

\[ T = \frac{1}{2} Ma - mg = -ma \]

\[ a = \frac{mg}{\left(\frac{1}{2} M + m\right)} \]

\[ T = \frac{1}{2} M \left(\frac{mg}{\frac{1}{2} M + m}\right) \]
Rotation about a moving axis

- The motion of an object can always be broken up into translation of the CoM and rotation of the CoM.

- So we can write the kinetic energy of the object as:

\[ K_{tot} = \frac{1}{2} Mv_{cm}^2 + \frac{1}{2} I_{cm} \omega^2 \]

- If we have rolling without slipping, then \( v_{cm} \) is related to \( \omega \):

\[
\theta = 0 \quad \theta = \pi/2 \quad \theta = \pi \quad \theta = 3\pi/2 \quad \theta = 2\pi
\]

\[ v_{cm} = \frac{\Delta s}{\Delta t} = \frac{2\pi}{\Delta t} R \quad \text{and} \quad \omega = \frac{\Delta \theta}{\Delta t} = \frac{2\pi}{\Delta t} \]

Condition for rolling w/o slipping:

\[ v_{cm} = \omega R \]
A block and a ball have the same mass and move with the same speed across a horizontal floor. The block slides without friction and the ball rolls without slipping.

Which one has the most kinetic energy?

a) the block
b) the ball
c) they have the same kinetic energy
A block and a ball have the same mass and start at rest at the top of identical ramps. The block slides down the ramp without friction and the ball rolls down the ramp without slipping.

Which one has the most kinetic energy at the bottom of the ramp?

a) the block
b) the ball

**c) they have the same kinetic energy**
A block and a ball have the same mass and move with the same initial velocity across a floor and then encounter identical ramps. The block slides without friction and the ball rolls without slipping.

Which one makes it furthest up the ramp?

a) Block

b) Ball

c) Both reach the same height.

Checkpoint: Rotational dynamics question 1

Block and Ball on Ramp: Question 1 (N = 85)
A cylinder and a hoop have the same mass and radius. They are released at the same time and roll down a ramp without slipping.

Which one reaches the bottom first?

a) Cylinder
b) Hoop
c) Both reach the bottom at the same time.
**Clicker Question** (Last seen as rotational dynamics question 3)

A small light cylinder and a large heavy cylinder are released at the same time and roll down a ramp without slipping.

Which one reaches the bottom first?
- a) Small cylinder
- b) Large cylinder
- c) Both reach the bottom at the same time.

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**Two Cylinders on Ramp: Question 1 (N = 85)**

<table>
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<tr>
<th>Option</th>
<th>% of Students</th>
</tr>
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<tbody>
<tr>
<td>A</td>
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<tr>
<td>B</td>
<td>37.6</td>
</tr>
<tr>
<td>C</td>
<td>44.7</td>
</tr>
</tbody>
</table>

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**Remember**

\[
\begin{align*}
|\mathbf{\tau}| &= F \cdot r \\
\sum \tau &= I \alpha \\
a_{\text{tan}} &= \alpha R \\
K_{\text{trans}} &= \frac{1}{2}mv^2 \\
K_{\text{rot}} &= \frac{1}{2}I\omega^2 \\
I_{\text{cyl}} &= \frac{1}{2}MR^2 \\
K_i + U_i + W_{\text{other}} &= K_f + U_f
\end{align*}
\]