Monday
- Potential Energy
- Gravitational PE and work
- Energy Conservation
- Friction and other work
- Elastic PE and work

Today
- Forces and potential energy functions
- Energy diagrams
- Examples
Prelecture question 2

If we define the potential energy in Case 1 to be zero, what is the potential energy of Case 2?

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\[ +x \text{ dir. where the spring expands} \]
\[ \overrightarrow{F_{\text{spring}}} = -k(x-x_0)t \]
\[ x_0 = -l \]
\[ +x \]
\[ \overrightarrow{F_{\text{spring}}} = -k(x+l)t \]

\[ \sum F_x = mg - k(x+l) = 0 \]
\[ mg = k(x+l) \quad x = 0 \]
\[ l = \frac{mg}{k} \]

\[ W_{\text{spring}} = \int_{0}^{x_0} -k(x+2l)dx \]
\[ = \left( -\frac{kx^2}{2} + klx \right) \bigg|_{0}^{x_0} = -\frac{kD^2}{2} + Dk \]

\[ W_{\text{grav}} = \int_{0}^{x_0} mg \, dx = -mgD \]

\[ W_{\text{tot}} = -\frac{kD^2}{2} + kDl - mgD \]

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10/15/2014
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A spring is compressed a distance \( d \) by a block of mass \( m \) which is initially at rest on the floor. The kinetic coefficient of friction between the floor and the block is \( \mu_k \). The mass is released and the spring launches it to the right, where it eventually stops a distance \( D \) from its starting point.

What is the macroscopic work done on the block by friction during this process?

a) \( W_{\text{friction}} = -\mu_k mgD \)

b) \( W_{\text{friction}} = \mu_k mgD \)

c) \( W_{\text{friction}} = 0 \)
A block of mass m, initially held at rest on a frictionless ramp a vertical distance H above the floor, slides down the ramp and onto a floor where friction causes it to stop a distance D from the bottom of the ramp. The coefficient of kinetic friction between the box and the floor is $\mu_k$.

What is the macroscopic work done on the block by friction during this process?

a) $mgH$

b) $-mgH$

c) $\mu_kmgD$

d) 0
A block of mass \( m \), initially held at rest on a frictionless ramp a vertical distance \( H \) above the floor, slides down the ramp and onto a floor where friction causes it to stop a distance \( D \) from the bottom of the ramp. The coefficient of kinetic friction between the box and the floor is \( \mu_k \).

What is the macroscopic work done on the block by all forces during this process?

a) \( mgH \)

b) \(-mgH\)

c) \( \mu_kmgD \)

d) 0
Clicker Question

A block is released from rest on a frictionless incline as shown. When the moving block is in contact with the spring and compressing it, what is happening to the gravitational potential energy $U_{\text{grav}}$ and the elastic potential energy $U_{\text{el}}$?

A. $U_{\text{grav}}$ and $U_{\text{el}}$ are both increasing.

B. $U_{\text{grav}}$ and $U_{\text{el}}$ are both decreasing.

C. $U_{\text{grav}}$ is increasing; $U_{\text{el}}$ is decreasing.

D. $U_{\text{grav}}$ is decreasing; $U_{\text{el}}$ is increasing.

E. The answer depends on how the block’s speed is changing.
Conservative forces and potential energy

We’ve been talking about “storing” potential energy to later be converted into kinetic. The forces that allow for the energy conversion between kinetic and potential are called conservative forces.

- Remember that when only conservative force are involved the total energy is conserved.

\[ E_{tot} = K_i + U_i = K_f + U_f \]

The work done by a conservative force will always have the following characteristics:

- It can be expressed as the difference between initial and final values of a potential energy function.
- It is reversible.
- It is independent of the path of the body, and depends only on the starting and ending points.
- When the starting and ending points are the same then total work done is zero.
Conservative forces and potential energy

For a conservative force we can write down a corresponding potential energy function

\[ W = -\Delta U \]

\[ F(x) \cdot \Delta x = -\Delta U \]

\[ F(x) = -\frac{\Delta U}{\Delta x} \]

\[ F(x) = -\lim_{\Delta x \to 0} \frac{\Delta U}{\Delta x} \]

- The force applied is equal to the negative slope of the potential energy.

- That means when the potential energy is changing rapidly (large slope) the magnitude of the force needed to do that is also large.

- When the force is in the \(+x\)-direction then \(U(x)\) is decreasing with increasing \(x\) so they have opposite signs.

A conservative force always pushes a body toward a lower potential energy.
Conservative forces and potential energy

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\[ F(x) = -\frac{dU}{dx} \]

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A conservative force always pushes a body toward a lower potential energy.
Examples of potential energy functions

Gravitational Potential energy

\[ U_{\text{grav}} = mg y \]

\[ \frac{d}{dy} (U_{\text{grav}} = mg y) \]

\[ \frac{dU}{dy} = mg \text{ in } y \text{-dir} \]

\[ F_{\text{grav},y} = -\frac{dU}{dy} = -mg \hat{j} \]

Elastic Potential energy

\[ U_{\text{el}} = \frac{1}{2} k x^2 \]

\[ \frac{d}{dx} (U_{\text{el}} = \frac{1}{2} k x^2) \]

\[ \frac{dU}{dx} = kx \]

\[ F_{\text{spring},x} = -kx \hat{i} \]

Expanding to 3D is straightforward, just do each piece separately and put everything together for the force vector.

\[ F_x = -\frac{dU}{dx} \]

\[ F_y = -\frac{dU}{dy} \]

\[ F_z = -\frac{dU}{dz} \]

Force vector for the potential function

\[ \vec{F} = \left( -\frac{dU}{dx} \right) \hat{i} + \left( -\frac{dU}{dy} \right) \hat{j} + \left( -\frac{dU}{dz} \right) \hat{k} \]
Examples of potential energy functions

Gravitational Potential energy

\[ U_{\text{grav}} = mgy \]
\[ \frac{dU}{dy} = \frac{d}{dy}(mgy) \]
\[ F_{\text{grav}} = -\frac{dU}{dy} \hat{j} = -mg\hat{j} \]

Elastic Potential energy

\[ U_{\text{el}} = \frac{1}{2}kx^2 \]
\[ \frac{dU}{dx} = \frac{d}{dx}\left(\frac{1}{2}kx^2\right) \]
\[ F_{\text{el}} = -\frac{dU}{dx} \hat{i} = -kx\hat{i} \]

Expanding to 3D is straightforward, just do each piece separately and put everything together for the force vector.

\[ F_x = -\frac{dU}{dx} \]
\[ F_y = -\frac{dU}{dy} \]
\[ F_z = -\frac{dU}{dz} \]

Force vector for the potential function

\[ \vec{F} = \left( -\frac{dU}{dx} \right)\hat{i} + \left( -\frac{dU}{dy} \right)\hat{j} + \left( -\frac{dU}{dz} \right)\hat{k} \]
Example 2D potential function

An object moving in the x-y plane is acted on by a conservative force described by the potential energy function \( U(x,y) = \alpha \left( \frac{1}{x^2} + \frac{1}{y^2} \right) \), where \( \alpha \) is a positive constant. Derive an expression for the force expressed in terms of the unit vectors \( \hat{i} \) and \( \hat{j} \)

\[
\vec{F} = -\frac{d}{dx} U(x,y) = -\frac{d}{dx} \left[ \alpha \left( \frac{1}{x^2} + \frac{1}{y^2} \right) \right] \hat{i} \\
= -\alpha \left( \frac{d}{dx} \left[ \frac{1}{x^2} \right] + \frac{d}{dx} \left[ \frac{1}{y^2} \right] \right) \hat{i} \\
= -\alpha \left( -\frac{2}{x^3} \right) \hat{i} \\
= -\alpha \left( -\frac{2}{x^3} \right) \hat{i}
\]

\[
\vec{F} = \frac{2\alpha}{x^3} \hat{i} + \frac{2\alpha}{y^3} \hat{j}
\]

Remember:

\[
\vec{F} = m\vec{a} \\
W = -\Delta U \\
\vec{F} = \left( \frac{dU}{dx} \right) \hat{i} + \left( \frac{dU}{dy} \right) \hat{j} + \left( \frac{dU}{dz} \right) \hat{k}
\]
The graph shows a conservative force $F_x$ as a function of $x$ in the vicinity of $x = a$. As the graph shows, $F_x > 0$ and $dF_x/dx < 0$ at $x = a$. Which statement about the associated potential energy function $U$ at $x = a$ is correct?

A. $dU/dx > 0$ at $x = a$

B. $dU/dx < 0$ at $x = a$

C. $dU/dx = 0$ at $x = a$

D. Any of the above could be correct.
Example Testing Conservative Forces with total work done

In a region of space the force on an electron is \( \vec{F} = Cx \hat{j} \), where \( C \) is a positive constant. The electron moves around a square loop in the xy-plane. Calculate the work done on the electron by the force during a counterclockwise trip around the square. Is this force conservative or not?

\[ W = \int F \cdot dl \]

**Leg 1**
\[ W_1 = \int_0^L Cx \hat{j} \cdot dx \hat{x} = 0 \]

**Leg 2**
\[ W_2 = \int_0^L Cx \hat{j} \cdot dy \hat{y} \]
\[ = CL \int_0^L dy = CLy \bigg|_0^L = CL^2 \]

**Leg 3**
\[ W_3 = 0 \]

**Leg 4**
\[ W_4 = \int_0^L Cx \hat{j} \cdot dy \hat{y} = 0 \]

\[ W_{tot} = W_1 + W_2 + W_3 + W_4 = CL^2 \]

Not Conservative
Example Testing Conservative Forces with total work done – Cont.

In a region of space the force on an electron is $\vec{F} = Cx\hat{j}$, where $C$ is a positive constant. The electron moves around a square loop in the xy-plane. Calculate the work done on the electron by the force during a counterclockwise trip around the square. Is this force conservative or not?
Energy Diagrams

Simple Spring

\[ U_{sp} = \frac{1}{2} kx^2 \]

\[ E = K + U \]

More General

Unstable equilibrium points are maxima in the potential-energy curve.

Stable equilibrium points are minima in the potential-energy curve.

\[ \frac{du}{dx} = 0 \quad F = 0 \]

\[ F = -\frac{dU}{dx} \]
Prelecture Question 2

Suppose the potential energy $U$ of some object as a function of $x$ looks like the plot shown below.
At which value of the $x$ values shown is the magnitude of the force on the object biggest?

![Plot of potential energy $U(x)$ vs. $x$]

\[
\frac{dU}{dx} \text{ largest value gives largest } |F|
\]

First Answer Choice Distribution ($N = 79$)

- A: 44.3%
- B: 27.8%
- C: 27.8%
Checkpoint Question 2

The potential energy of an object U as a function of x looks like the plot shown above. Where is the force the biggest in the negative x direction?

\[ U(x) \]

Where \(-F\) is largest

\[ \frac{dU}{dx} \text{ is largest} \]

(a) (b) (c) (d)

Potential Energy Function: Question 1 (N = 77)

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<tr>
<td>D</td>
<td>1.3</td>
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</table>
A particle moving along the x-axis is acted on by a conservative force $F_x$ described by the potential energy function shown. At point b which of the following can be said about the particle:

a) It is experiencing no force

b) It is experiencing a force in the +x direction

c) It is at a stable equilibrium point

d) It is at an unstable equilibrium point

e) Both (a) and (c)
Example Acceleration from potential function

A small block with mass 0.04 kg is moving in the x-y plane. The net force on the block is described by the potential $U(x,y) = (5.80 \text{ J/m}^2) x^2 - (3.60 \text{ J/m}^3) y^3$. What is the acceleration of the block when it is at the point $x = 0.300 \text{ m}, y = 0.600 \text{ m}$?

$$
\vec{F} = m\vec{a} \\
F_x = -\frac{dU}{dx} = -d(5.80x^2) + d(3.60y^3) \\
F_x = -2(5.80)x
$$

$$
\vec{F} = m\vec{a} = -11.6x\hat{i} + 10.8y^2\hat{j} \\
\vec{a} = \frac{\vec{F}}{m} = -290x\hat{i} + 270y^2\hat{j} \quad [\text{m/s}^2] \\
\vec{a}(0.300\text{ m}, 0.600\text{ m}) = -87\hat{i} + 97.2\hat{j} \quad [\text{m/s}^2]
$$

Remember:

$$
\vec{F} = m\vec{a} \\
W = -\Delta U \\
\vec{F} = \left(-\frac{dU}{dx}\right)\hat{i} + \left(-\frac{dU}{dy}\right)\hat{j} + \left(-\frac{dU}{dz}\right)\hat{k}
$$
Example Rock up a hill (eoc 7.48)

A 28kg rock approaches the foot of a hill with a speed of 15 m/s. This hill slopes upward at a constant angle of 40° above the horizontal. The coefficients of static and kinetic friction between the rock and hill are 0.75 and 0.20.

a) Use energy conservation to find the maximum height reached by the rock above the foot of the hill.
b) Will the rock remain at rest at its highest point, or will it slide back down?
c) If it slides back down how fast will its final speed be when it reaches the bottom of the hill?
Example Rock up a hill (eoc 7.48) – cont.

b) Will the rock remain at rest at its highest point, or will it slide back down?
c) If it slides back down how fast will its final speed be when it reaches the bottom of the hill?
A 350 kg roller coaster starts from rest at point A and slides down the frictionless loop-the-loop shown.

a) How fast is the roller coaster moving at point B?
b) How hard does it press against the track at point B?

\[ K_A = 0 \]
\[ U_A = mgh_A = (350\text{kg})(9.8\text{m/s}^2)(25\text{m}) \]
\[ K_B = \frac{1}{2}mv^2 \]
\[ U_B = mgh_B \]
\[ K_A + U_A = K_B + U_B \]
\[ \frac{1}{2}(mgh_A - mgh_B) = \frac{1}{2}mv^2 \]

\[ S^f y = -mg - n = - \frac{mv^2}{R} \]
\[ n = \frac{mv^2}{R} - mg \]
\[ n = 1.02 \times 10^6 \text{N} \]

Remember:

\[ K_i + U_i + W_{other} = K_f + U_f \]
\[ U_{grav} = mgy \]
\[ K = \frac{1}{2}mv^2 \]
A cutting tool under a microprocessor control has several forces acting on it. One force is \( \vec{F} = -\alpha xy^2 \hat{j} \) (in the \(-y\)-direction and dependent on the position), \( \alpha = 2.5 \text{N/m}^3 \).

Consider the displacement of the tool from the origin to the point \( x = 3.00 \text{m}, y = 3.00 \text{m} \)

a) Calculate the work done on the tool by \( F \) if this displacement is along the straight line \( y = x \) that connects these two points.

b) Calculate the work done on the tool by \( F \) if the tool is first moved out along the \( x \)-axis to \((3.00 \text{m}, 0 \text{m})\) and then moved up the \( y \)-axis to \((3.00 \text{m}, 3.00 \text{m})\)

c) Compare the work done by \( F \) along these two paths. Is \( F \) conservative or non-conservative?
**Example** both elastic and gravitational potential energy

A 2000kg (19,600N) elevator with broken cables in a test rig is falling at 4.00m/s when it contacts a cushioning spring at the bottom of the shaft. The spring is intended to stop the elevator, compressing 2.00m as it does. During the motion a safety clamp applies 17,000N frictional force to the elevator. What is the necessary force constant \( k \) of the spring?

\[
K_i + U_i + W_{\text{other}} = K_f + U_f
\]

\[
U_{\text{grav}} = mgy
\]

\[
U_{\text{el}} = \frac{1}{2}kx^2
\]

\[
K = \frac{1}{2}mv^2
\]

\[
W = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}
\]
Example both elastic and gravitational potential energy

Remember:

\[ K_i + U_i + W_{other} = K_f + U_f \]

\[ U_{grav} = mg y \]

\[ U_{el} = \frac{1}{2} k x^2 \]

\[ K = \frac{1}{2} m v^2 \]

\[ W = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r} \]
The potential energy of an object U as a function of x looks like the plot shown. Where is the force the biggest in the positive x direction?

- (a) none, the force is constant over all of x