Chapter 8

Momentum, Impulse and Collisions
Learning Goals

- The meaning of the momentum of a particle(system) and how the impulse of the net force acting on a particle causes the momentum to change.
- The conditions under which the total momentum of the system of particles is constant (conserved).
- How to solve two-body collision problems.
- The distinction between elastic, inelastic and completely inelastic collisions.
- The definition of the center of mass of a system and what determines how the center of mass moves.
- Lastly, a very brief introduction to rocket propulsion.
I got confused in the math for finding the center of mass when taking into consideration all the particles, with the triple integral.

Center of mass in relation with the $x, y, z$ plane.

The difference between discrete distribution and continuous

All the integrating stuff,

a little bit of everything

how do you find the center of mass on a non uniform object. For instance the chess piece.

I'm not clear on how all the equations are derived or related to Newton's Third Law. Please go over the "Two Men Pulling" checkpoint questions.

i don't really understand this topic much

Do we have to memorize all of these equations?

Could you please discuss the center of mass equation more in depth and briefly go over the checkpoint questions in class.

Systems of two masses

the movement of center of mass when objects accelerate
What causes energy loss during collisions and how does this relate to the internal forces of the system? I still do not quite understand the relation between momentum and center of mass.

If you could go over practice problems that would be very helpful.

Transformation of CM frame

Please go over the second question of the "Cart Propulsion" checkpoint and more detail on the CM reference frame along with the equation that's similar to relative velocity.

Please go over center of mass reference farm. Also, energy lost in collision.

The loss of energy. Namely the difference between kinetic energy and momentum that allows the first to change and not the latter.
Not so many equations if you use these as your tool kit.
Definition of Momentum

(revisited)

\[ \sum \vec{F}_{Net} = m \frac{d\vec{v}}{dt} = \frac{d}{dt} (m\vec{v}) \]

we defined the quantity \( m\vec{v} \)

\( \vec{p} = m\vec{v} \) as the momentum of the particle

\[ \sum \vec{F}_{Net} = \frac{d\vec{p}}{dt} \]

as an alternate form of Newton's 2nd Law
The Definition of Impulse

\[ \vec{J} = \sum \vec{F}_{\text{net}} (t_2 - t_1) = \sum \vec{F} \Delta t \]

assuming a constant net force

\[ \sum \vec{F}_{\text{net}} = \frac{\vec{p}_2 - \vec{p}_1}{t_2 - t_1} \]

substituting for the impulse, this becomes

\[ \sum \vec{F}_{\text{Net}} (t_2 - t_1) = \vec{p}_2 - \vec{p}_1 \]

\[ \vec{J} = \vec{p}_2 - \vec{p}_1 \]
For forces that are not constant the impulse becomes

\[ \vec{J} = \int_{t_1}^{t_2} \sum F_{net} \, dt = \vec{p}_2 - \vec{p}_1 \]

general definition of an impulse (impulse is a vector and has units of newton - seconds or kg m/s)
Figure 8.3

(a) The area under the curve of net force versus time equals the impulse of the net force:

\[ \text{Area} = J_x = \int_{t_1}^{t_2} \Sigma F_x \, dt \]

We can also calculate the impulse by replacing the varying net force with an average net force:

\[ \text{Area} = \int_{t_1}^{t_2} (F_{av})_x \, dt = (F_{av})_x (t_2 - t_1) \]

(b) The area under both curves is the same, so both forces deliver the same impulse.

Large force that acts for a short time

Smaller force that acts for a longer time
Collisions and Impulse!

- http://www.youtube.com/watch?v=W9EqU1_DXUw

Do not do this at home!!
A ball (mass 0.40 kg) is initially moving to the left at 30 m/s. After hitting the wall, the ball is moving to the right at 20 m/s. What is the impulse of the net force on the ball during its collision with the wall?

A. 20 kg \cdot m/s to the right
B. 20 kg \cdot m/s to the left
C. 4.0 kg \cdot m/s to the right
D. 4.0 kg \cdot m/s to the left
E. none of the above
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A. 20 kg • m/s to the right
B. 20 kg • m/s to the left
C. 4.0 kg • m/s to the right
D. 4.0 kg • m/s to the left
E. none of the above
You are testing a new car using crash test dummies. Consider two ways to slow the car from 90 km/h (56 mi/h) to a complete stop:

(i) You let the car slam into a wall, bringing it to a sudden stop.

(ii) You let the car plow into a giant tub of gelatin so that it comes to a gradual halt.

In which case is there a greater impulse of the net force on the car?

A. in case (i)
B. in case (ii)
C. The impulse is the same in both cases.
D. not enough information given to decide
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(i) You let the car slam into a wall, bringing it to a sudden stop.
(ii) You let the car plow into a giant tub of gelatin so that it comes to a gradual halt.

In which case is there a greater net stopping force on the car?

A. in case (i)
B. in case (ii)
C. The stopping force is the same is the same in both cases.
D. not enough information given to decide
Conservation of Momentum

If the vector sum of the external forces on a system is zero, the total momentum of the system is conserved.

\[ \sum \vec{F}_{\text{Net}} = 0 = \frac{d\vec{p}}{dt} \]

and the momentum of the system is constant.
For a two body system with no external forces acting.

\[ \vec{F}_{BonA} = \frac{d\vec{p}_A}{dt} \; ; \; \vec{F}_{AonB} = \frac{d\vec{p}_B}{dt} \]

\[ \vec{F}_{BonA} + \vec{F}_{AonB} = \frac{d\vec{p}_A}{dt} + \frac{d\vec{p}_B}{dt} = 0 \]

\[ \frac{d(\vec{p}_A + \vec{p}_B)}{dt} = 0 \]

so the total momentum is constant.
Suppose you are on a cart, initially at rest, which rides on a frictionless horizontal track. If you throw a ball off the cart towards the left, will the cart be put into motion?

- Yes, and it moves to the right.
- Yes, and it moves to the left.
- No, it remains in place.
Suppose you are on a cart, initially at rest, which rides on a frictionless horizontal track. You throw a ball at a vertical surface that is firmly attached to the cart. If the ball bounces straight back as shown in the picture, will the cart be put into motion after the ball bounces back from the surface?

- Yes, and it moves to the right.
- Yes, and it moves to the left.
- No, it remains in place.
Recoil of a rifle

Before

Rifle + bullet

After

\[ v_{Rx} = ? \]

\[ m_R = 3.00 \text{ kg} \]

\[ v_{Bx} = 300 \text{ m/s} \]

\[ m_B = 5.00 \text{ g} \]
(a) Before collision

(b) Collision

(c) After collision

\[ v_{A1x} = 2.0 \text{ m/s} \quad v_{B1x} = -2.0 \text{ m/s} \]

\[ m_A = 0.50 \text{ kg} \quad m_B = 0.30 \text{ kg} \]

\[ v_{A2x} \quad v_{B2x} = 2.0 \text{ m/s} \]
Elastic collisions in 1-D

\[ \vec{p}_{\text{total\,-\,before}} = m_A \vec{v}_{A1} \]

\[ \vec{p}_{\text{total\,-\,after}} = m_A \vec{v}_{A2} + m_B \vec{v}_{B2} \]

\[ KE_{\text{before}} = \frac{1}{2} m_A v_{A1}^2 \]

\[ KE_{\text{after}} = \frac{1}{2} m_A v_{A2}^2 + \frac{1}{2} m_B v_{B2}^2 \]

We can solve these equations for the velocities after the collision

\[ v_{A2} = \frac{m_A - m_B}{m_A + m_B} v_{A1} \quad \text{and} \]

\[ v_{B2} = \frac{2m_A}{m_A + m_B} v_{A1} \]
Elastic collisions

http://www.youtube.com/watch?v=mFNe_pFZrsA