Announcements

• Chapter 9 HW due Wed Nov 8th
• Chapter 10 HW due Monday Nov 13th, as usual

• First Announcement of Exam 3:
  – November 21st
  – Tuesday before Thanksgiving!
Rotational Motion

Chapters 9 and 10 in four lectures

• Lecture three of the four lectures

• Concentrate on the relationship between linear and angular variables

• Today: Finish up topics

• Thursday: Hard problems
Angular Quantities

• Position \( \rightarrow \) Angle \( \theta \)
• Velocity \( \rightarrow \) Angular Velocity \( \omega \)
• Acceleration \( \rightarrow \) Angular Acceleration \( \alpha \)
• Force \( \rightarrow \) Torque \( \tau \)
• Mass \( \rightarrow \) Moment of Inertia \( I \)

Today we’ll finish:

– Momentum
– Energy
Momentum

Momentum vs. Angular Momentum:

\[ \vec{p} = m\vec{v} \rightarrow \vec{L} = I\vec{\omega} \]

Newton’s Laws:

\[ \vec{F} = \frac{d\vec{p}}{dt} \rightarrow \vec{\tau} = \frac{d\vec{L}}{dt} \]
Angular Momentum

First way to define the Angular Momentum $L$:

\[ \vec{L} = I \vec{\omega} \]

\[ \sum \vec{\tau} = I \vec{\alpha} = I \frac{d\vec{\omega}}{dt} = \frac{d(I\vec{\omega})}{dt} = \frac{d(L)}{dt} = \frac{d\vec{L}}{dt} \]

\[ \sum \vec{\tau} = I \vec{\alpha} = \frac{d\vec{L}}{dt} \]
Angular Momentum Definition

Another definition:

$$\vec{L} = \vec{r} \times \vec{p}$$
Angular Motion of a Particle

Determine the angular momentum, \( L \), of a particle, with mass \( m \) and speed \( v \), moving in circular motion with radius \( r \).
Conservation of Angular Momentum

\[ \sum \vec{\tau} = \frac{d \vec{L}}{dt} \]

if \( \sum \tau = 0 \rightarrow L = \text{Const} \)

By Newton’s laws, the angular momentum of a body can change, but the angular momentum for a system cannot change.

Conservation of Angular Momentum

Same as for linear momentum
Ice Skater

• This one you’ve seen on TV
• Try this at home in a chair that rotates
• Get yourself spinning with your arms and legs stretched out, then pull them in

\[ \vec{L} = I \vec{\omega} \]

- \( I \) large, \( \omega \) small
- \( I \) small, \( \omega \) large
Problem Solving

For Conservation of Angular Momentum problems:

BEFORE and AFTER
Conservation of Angular Momentum

Before

Toast always lands butter Side down.

Attach to back of.....

...Cat Always lands on feet.
After
As a car engineer, you model a car clutch as two plates, each with radius $R$, and masses $M_A$ and $M_B$ ($I_{\text{Plate}} = \frac{1}{2}MR^2$). Plate $A$ spins with speed $\omega_1$ and plate $B$ is at rest. You close them so they spin together.

Find the final angular velocity of the system.
Angular Quantities

• Position $\rightarrow$ Angle $\theta$
• Velocity $\rightarrow$ Angular Velocity $\omega$
• Acceleration $\rightarrow$ Angular Acceleration $\alpha$
• Force $\rightarrow$ Torque $\tau$
• Mass $\rightarrow$ Moment of Inertia $I$

Today we’ll finish:

– Momentum $\rightarrow$ Angular Momentum $L$
– Energy $\leftarrow$
Rotational Kinetic Energy

\[ KE_{\text{trans}} = \frac{1}{2}mv^2 \]

\[ \rightarrow KE_{\text{rotate}} = \frac{1}{2}I\omega^2 \]

Conservation of Energy must take rotational kinetic energy into account
Rotation and Translation

- **Objects can both **Rotate and Translate**

- **Need to add the two**

\[ KE_{total} = \frac{1}{2} mv^2 + \frac{1}{2} I \omega^2 \]

- **Rolling without slipping is a special case where you can relate the two**

\[ V = \omega r \]
Rolling Down an Incline

You take a solid ball of mass $m$ and radius $R$ and hold it at rest on a plane with height $Z$. You then let go and the ball rolls without slipping.

What will be the speed of the ball at the bottom?

What would be the speed if the ball didn’t roll and there were no friction?

Note: $I_{\text{sphere}} = \frac{2}{5}MR^2$
A bullet of speed $V$ and mass $m$ strikes a solid cylinder of mass $M$ and inertia $I = \frac{1}{2}MR^2$, at radius $R$ and sticks. The cylinder is anchored at point 0 and is initially at rest.

What is $\omega$ of the system after the collision?

Is energy Conserved?
Rotating Rod

A rod of mass uniform density, mass $m$ and length $l$ pivots at a hinge. It has moment of inertia $I=ml/3$ and starts at rest at a right angle. You let it go:

What is $\omega$ when it reaches the bottom?

What is the velocity of the tip at the bottom?
A heavy pulley, with radius \( R \), starts at rest. We pull on an attached rope with constant force \( F_T \). It accelerates to final angular speed \( \omega \) in time \( t \).

A better estimate takes into account that there is friction in the system. This gives a torque (due to the axel) we’ll call this \( \tau_{\text{fric}} \).

What is this better estimate of the moment of Inertia?
Person on a Disk

A person with mass $m$ stands on the edge of a disk with radius $R$ and moment $\frac{1}{2}MR^2$. Neither is moving.

The person then starts moving on the disk with speed $V$.

Find the angular velocity of the disk.
Same Problem: Forces

Same problem but with Forces

\[ \begin{align*}
F_{fr} & \quad Mg \cos \theta \\
Mg & \quad Mg \sin \theta \\
F_N & \quad Mg
\end{align*} \]

\[ H \]

\[ \theta \]
Next Time

• More problems on Chapters 9 & 10
  – Please get caught up on homework!!!
  – Believe it or not exam 3 is just around the bend, Nov 21st
  – Tuesday before Thanksgiving!

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• Chap 10 HW due Nov 13th