Before we begin

- Chapter 8 HW was due Monday
- Ch. 10 reading assignment: due today
- Already covered most of chapter 9 in lecture, but I’m changing the Chapter 9 HW due date to next Wednesday
- Ch. 10 HW still due the following Monday
Rotational Motion

Chapters 9 and 10 in four combined lectures

• This is the 2\textsuperscript{nd} of the 4 lectures

• Concentrate on the relationship between \textit{linear} and \textit{angular} variables

• Last time started kinematics… Move to dynamics just like earlier this semester
Angular Quantities

Last time:

- Position $\Rightarrow$ Angle $\theta$
- Velocity $\Rightarrow$ Angular Velocity $\omega$
- Acceleration $\Rightarrow$ Angular Acceleration $\alpha$

This time we’ll start by discussing the vector nature of the variables and then move forward on the others:

- Force
- Mass
- Momentum
- Energy
Angular Velocity and Acceleration

Are $\omega$ and $\alpha$ vectors?

$\omega$ and $\alpha$ clearly have magnitude

Do they have direction?
Yes!

Define the direction to point along the axis of rotation

Right-hand Rule

This is true for Θ, ω and α
Angular Quantities

- Position $\rightarrow$ Angle $\theta$
- Velocity $\rightarrow$ Angular Velocity $\omega$
- Acceleration $\rightarrow$ Angular Acceleration $\alpha$

Moving forward:

- Force
- Mass
- Momentum
- Energy
Torque

- Torque is the analogue of Force
- Take into account the perpendicular distance from axis
  - Same force further from the axis leads to more Torque
Slamming a door

We know this from experience:

– If we want to slam a door really hard, we grab it at the end

– If we try to push in the middle, we aren’t able to make it slam nearly as hard
Torque Continued

• What if we change the angle at which the Force is applied?

• What is the “Effective Radius?”
Slamming a door

We also know this from experience:

– If we want to slam a door really hard, we grab it at the end and “throw” perpendicular to the hinges

– If we try to pushing towards the hinges, the door won’t even close
Torque

• Torque is our “slamming” ability

• Write Torque as $\tau$

\[
| \vec{\tau} | = r \parallel F \sin \theta
\]

\[
\vec{\tau} = \vec{r} \times \vec{F}
\]

• To find the direction of the torque, wrap your fingers in the direction the torque makes the object twist
Torque and Force

Torque problems are like Force problems

1. Draw a force diagram
2. Then, sum up all the torques to find the total torque

Is torque a vector?
Example: Composite Wheel

Two forces, $F_1$ and $F_2$, act on different radii of a wheel, $R_1$ and $R_2$, at different angles $\Theta_1$ and $\Theta_2$. $\Theta_1$ is a right angle.

If the axis is fixed, what is the net torque on the wheel?
Angular Quantities

- Position $\rightarrow$ Angle $\theta$
- Velocity $\rightarrow$ Angular Velocity $\omega$
- Acceleration $\rightarrow$ Angular Acceleration $\alpha$

Moving forward:

- Force $\rightarrow$ Torque $\tau$
- Mass
- Momentum
- Energy
The analogue of Mass is called **Moment of Inertia**

Example: A ball of mass $m$ moving in a circle of radius $R$ around a point has a moment of inertia

$$F = ma \Rightarrow \tau = I \alpha$$
Calculate Moment of Inertia

Calculate the moment of inertia for a ball of mass $m$ relative to the center of the circle $R$. 
Moment of Inertia

- To find the mass of an object, just add up all the little pieces of mass.

- To find the moment of inertia around a point, just add up all the little moments.

\[ I = \sum mr^2 \quad \text{or} \quad I = \int r^2 dm \]
Torque and Moment of Inertia

• Force vs. Torque

\[ F = ma \Rightarrow \tau = I \alpha \]

• Mass vs. Moment of Inertia

\[ m \Rightarrow I = \sum mr^2 \quad \text{or} \quad I = \int r^2 dm \]
A heavy pulley, with radius $R$, and known moment of inertia $I$ starts at rest. We attach it to a bucket with mass $m$. The friction torque is $\tau_{f\text{ric}}$.

Find the angular acceleration $\alpha$
A heavy pulley, with radius $R$, starts at rest. We pull on an attached rope with a constant force $F_T$. It accelerates to an angular speed of $\omega$ in time $t$.

What is the moment of inertia of the pulley?
A heavy pulley, with radius $R$, starts at rest. We pull on an attached rope with constant force $F_T$. It accelerates to final angular speed $\omega$ in time $t$.

A better estimate takes into account that there is friction in the system. This gives a torque (due to the axel) we’ll call this $\tau_{\text{fric}}$.

What is this better estimate of the moment of Inertia?
Next Time

• The rest of Chapter 10
  – More on angular “Stuff”
  – Angular Momentum
  – Energy

• Get caught up on your homework!!!

• Chapter 8 was due Monday, Chapter 9 homework due Wednesday, Chapter 10 is due the Monday after that