Rotational Motion

Will do Chapters 9 and 10 in four combined lectures

• Start with *Fixed Axis* motion
• The relationship between *linear* and *angular* variables
• Rotating and translating at the same time
• First kinematics, then dynamics
  – just like earlier this semester
Overview: Rotational Motion

• Take our results from “linear” physics and do the same for “angular” physics

• We’ll discuss the analogue of
  – Position
  – Velocity
  – Acceleration
  – Force
  – Mass
  – Momentum
  – Energy
Rotational Motion

• Here we’re talking about stuff that goes around and around

• Start by envisioning:

A spinning object like a car tire
Some Buzz Phrases

• **Fixed axis**: *I.e.*, an object spins in the same place... an ant on a spinning top goes around the same place over and over again

  Another example: Earth has a fixed axis, the sun

• **Rigid body**: *I.e.*, the objects don’t change as they rotate. Example: a bicycle wheel

Examples of Non-rigid bodies?
Overview: Rotational Motion

- Take our results from "linear" physics and do the same for "angular" physics
- Analogue of
  - Position ←
  - Velocity ←
  - Acceleration ←
  - Force
  - Mass
  - Momentum
  - Energy

Start here!

Chapters 1-3
Axis of Rotation: Definitions

Pick a simple place to rotate around

Call point O the "Axis of Rotation"

Same as picking an origin
An Important Relation: Distance

If we are sitting at a radius $R$ relative to our axis, and we rotate through an angle $\theta$, then we travel through a distance $l$.

$l = \theta R$

$Circ = 2\pi R$
Velocity and Acceleration

Define $\omega$ as the angular velocity

$$\omega = \frac{\Delta \theta}{\Delta t} \quad \text{or} \quad \omega = \frac{d\theta}{dt} \quad \text{radians/sec}$$

Define $\alpha$ as the angular acceleration

$$\alpha = \frac{d\omega}{dt} \quad \text{or} \quad \alpha = \frac{d^2\theta}{dt^2} \quad \text{radians/sec}^2$$
What is the \textit{linear} speed of a point rotating around in a circle with angular speed $\omega$, and constant radius $R$?
Examples

Consider two points on a rotating wheel. One on the inside (P) and the other at the end (b):

• Which has greater angular speed?
• Which has greater linear speed?
Uniform Angular Acceleration

Derive the angular equations of motion for \textit{constant} angular acceleration

\[ \Theta = \Theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \]

\[ \omega = \omega_0 + \alpha t \]
Rotation and Translation

Objects can both translate and rotate at the same time. They do both around their *center of mass*. 
Rolling without Slipping

• In reality, car tires both rotate and translate

• They are a good example of something which rolls (translates, moves forward, rotates) without slipping

• Is there friction? What kind?
Derivation

• The trick is to pick your reference frame correctly!

• Think of the wheel as sitting still and the ground moving past it with speed $V$.

Velocity of ground (in bike frame) = $-\omega R$

$\Rightarrow$ Velocity of bike (in ground frame) = $\omega R$
Bicycle comes to Rest

A bicycle with initial linear velocity $V_0$ (at $t_0=0$) decelerates uniformly (without slipping) to rest over a distance $d$. For a wheel of radius $R$:

a) What is the angular velocity at $t_0=0$?
b) Total revolutions before it stops?
c) Total angular distance traversed by the wheel?
d) The angular acceleration?
e) The total time until it stops?
A computer hard drive typically rotates at 5400 rev/minute

Find the:

• Angular Velocity in $\text{rad/sec}$
• Linear Velocity on the rim ($R=3.0\,\text{cm}$)
• Linear Acceleration

It takes 3.6 sec to go from rest to 5400 rev/min, with constant angular acceleration.

• What is the angular acceleration?
Show for constant acceleration that:

\[ \omega_f^2 - \omega_0^2 = 2\alpha \Delta \theta \]
Exam 2

• Class average for the 2\textsuperscript{nd} exam (including the 5 points) was 80.1\%
  – Average for first two exams is a 78.7\%
• Straight scale for curve for now
• Many have asked “\textit{should I q-drop?}”
  – Talk to your advisor and read my FAQ!
  – Generic advice: \textit{Drop if you can’t keep up with the homework by yourself}
Next Time

• Lecture on Thursday will cover Chapters 9 and 10:
  – Reading questions due: Q10.7 & Q10.26
End of Lecture Notes