Checklist for Today

• Things that were due last Thursday:
  – Chapter 1 reading
  – Read all handouts from web page

• Things that are due yesterday (Monday):
  – WebCT warm-ups (FCI, Math Assess, etc…)
  – Math Quizzes 1 through 10

• Things that are due today:
  – Reading for Chapter 2
  – Chapter 2 Lecture Questions

• For this week and/or due next Monday:
  – Recitation: Lab materials, start Ch. 1 on WebCT
  – All HW1 problems on WebCT due Monday
Describing Motion

Interested in two key ideas:

• *How* objects move as a function of time
  – *Kinematics*
  – Chapters 2 and 3

• *Why* objects move the way they do
  – *Dynamics*
  – Do this in Chapters 4 and 5
Chapter 2: Motion in 1-Dimension

• Today: Velocity & Acceleration
  – Equations of Motion
  – Some calculus (derivatives)

• Thursday:
  – More calculus (integrals)
  – Problems
Notes before we begin

• This chapter is a good example of a set of material that is best learned by doing examples

• We’ll do some examples today

• Lots more next time...
Equations of Motion

We want **Equations** that describe

- *Where* am I as a function of time?
- *How fast* am I moving as a function of time?
- *What direction* am I moving as a function of time?
- Is its velocity *changing*? Etc.
Motion in One Dimension

- Where is the car?
  - $X=0$ feet at $t_0=0$ sec
  - $X=22$ feet at $t_1=1$ sec
  - $X=44$ feet at $t_2=2$ sec

- Since the car’s position is changing (i.e., moving) we say this car has “velocity” or “speed”

- Plot position vs. time
  - How do we get the velocity from the graph?
Velocity

Questions:

- How fast is my position changing?
- What would my speedometer read?
- What is my instantaneous Velocity?
How do we Calculate Velocity?

- Define Velocity: “Change in position during a certain amount of time”
- Math: Calculate from the Slope: The “Change in position as a function of time”
  - Change in Vertical divided by the Change in Horizontal
  - Velocity = \( \Delta X / \Delta t \)
Constant Velocity

Equation of Motion for this example is a straight line

Write this as:

\[ X = bt \]

- Slope is constant
- Velocity is constant
  - Easy to calculate
  - Same everywhere
Moving Car

A harder example:

\[ X = ct^2 \]

• What’s the velocity at \( t=1 \) sec?

Want to calculate the “Slope” here
Math Digression: Derivatives

- To find the slope at time $t$, just take the “derivative”
- For $X=ct^2$, $Slope = V = \frac{dx}{dt} = 2ct$
- “Gerbil” derivative method
  - If $X = at^n \rightarrow V = \frac{dx}{dt} = nat^{n-1}$
  - “Derivative of $X$ with respect to $t$”
- More examples
  - $X = qt^2 \rightarrow V = \frac{dx}{dt} = 2qt$
  - $X = ht^3 \rightarrow V = \frac{dx}{dt} = 3ht^2$
Common Mistakes

The trick is to remember what you are taking the derivative “with respect to”

More Examples (with \( a=\text{constant} \)):

- **What if \( X=2a^3t^n \)?**
  - Why not \( \frac{dx}{dt} = 3(2a^2t^n) \)?
  - Why not \( \frac{dx}{dt} = 3n(2a^2t^{n-1}) \)?

- **What if \( X=2a^3 \)?**
  - What is \( \frac{dx}{dt} \)?
  - There are no \( t \)'s!!! \( \frac{dx}{dt} = 0 \)!!!
  - If \( X=22 \text{ feet} \), what is the velocity? =0!!!
Check: Constant Position

- \( X = C = 22 \text{ feet} \)
- \( V = \text{slope} = dx/dt = 0 \)

- Check
Check: Constant Velocity

- **Car is moving**
  - \( X=0 \text{ feet at } t_0=0 \text{ sec} \)
  - \( X=22 \text{ feet at } t_1=1 \text{ sec} \)
  - \( X=44 \text{ feet at } t_2=2 \text{ sec} \)

- **What is the equation of motion?**
  - \( X = bt \) with \( b=22 \text{ ft/sec} \)
  - \( V = dX/dt \)
  - \( \vec{V} = b = 22 \text{ ft/sec} \)

- **Check**
Check: Non-Constant Velocity

- $X = ct^2$ with $c=11 \text{ ft/sec}^2$
- $V = \frac{dX}{dt} = 2ct$

- The velocity is:
  - “non-Constant”
  - a “function of time”
  - “Changes with time”
  - $V=0 \text{ ft/s}$ at $t_0=0 \text{ sec}$
  - $V=22 \text{ ft/s}$ at $t_1=1 \text{ sec}$
  - $V=44 \text{ ft/s}$ at $t_2=2 \text{ sec}$
Acceleration

• If your velocity is changing, you are “accelerating”
  – You hit the *accelerator* in your car to speed up at a stop light
    • (Ok…It’s true you also hit it to stay at constant velocity, but that’s because friction is slowing you down…we’ll get to that later…)

• How quickly is the velocity changing? That’s our **Acceleration**
Acceleration

- Acceleration is the “Rate of change of velocity”
- Said differently: “How fast is the Velocity changing?” “What is the change in velocity as a function of time?”

\[
\text{Accel} = \frac{\Delta V}{\Delta t} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{dV}{dt}
\]
Example

You have an equation of motion where:

\[ X = X_0 + V_0 t + \frac{1}{2} a t^2 \]

where \( X_0 \), \( V_0 \), and \( a \) are constants.

What is the velocity and the acceleration?

\[ V = \frac{dx}{dt} = 0 + V_0 + at \]

• Remember that the derivative of a constant is Zero!!

\[ \text{Accel} = \frac{dV}{dt} = \frac{d^2x}{dt^2} = 0 + 0 + a \]
Position, Velocity and Acceleration

• All three are related
  – Velocity is the *derivative* of position with respect to time
  – Acceleration is the *derivative* of velocity with respect to time
  – Acceleration is the *second derivative* of position with respect to time

• Calculus is REALLY important

• Derivatives are something we’ll come back to over and over again
Important Equations of Motion

If the acceleration is constant

\[ \vec{v} = \vec{v}_0 + \vec{a}t \]

\[ \vec{x} = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a}t^2 \]

Position, velocity and Acceleration are vectors. More on this in Chap 3
Conceptual Example

• If the velocity of an object is zero, does it mean that the acceleration is zero?

• If the acceleration is zero, does that mean that the velocity is zero?
Car Crash Test Design

You are designing a crash test setup for a car maker. You can accelerate a car from rest with a constant acceleration of 1.00 m/s² so you can make the car crash into a wall. (This is the last time you will see numbers in a problem in lecture).

1. If the path is 200m long, what is the velocity of the car just before/as it hits the wall?

2. For the same acceleration, if you want the car to hit the wall with a speed of 30m/s (about 60 mi/hr), what minimum length must you have?