Gravity is everywhere

1. Matter is pulled from the ordinary star to form an accretion disk around the black hole.

2. The gas in the accretion disk is compressed and heated to high temperatures, becoming an intense source of x rays.

3. Gas in the accretion disk that does not fall into the black hole is ejected in two fast-moving jets.
Gravitation
(due to Newton)

\[ g = 9.8 \frac{m}{s^2} \]

\[ r = 3.84 \times 10^8 \text{ m} \]
\[ T = 27.3 \text{ days} \approx 2.36 \times 10^6 \text{ s} \]

\[ a = \frac{v^2}{r} = \left( \frac{2\pi r}{T} \right)^2 \frac{1}{r} = \frac{4\pi^2 r}{T^2} = 2.7 \times 10^3 \frac{m}{s^2} = 2.8 \times 10^4 \frac{g}{3600} \]

\[ \frac{r}{R_E} = 60 \quad a \sim \frac{1}{r^2} \]

\[ F = m a \sim \frac{mM}{r^2} \]

\[ F = G \frac{mM}{r^2} \]
In 1798 Henry Cavendish made his sensitive measurement to determine a numerical value for the constant $G$.

1. Gravitation pulls the small masses toward the large masses, causing the vertical quartz fiber to twist.

   The small balls reach a new equilibrium position when the elastic force exerted by the twisted quartz fiber balances the gravitational force between the masses.

2. The deflection of the laser beam indicates how far the fiber has twisted. Once the instrument is calibrated, this result gives a value for $G$. 

   ![Diagram of the Cavendish experiment](image)
Gravity decreases as altitude rises

Earth, mass $m_E$

Earth’s radius $R_E = 6.38 \times 10^6$ m

Astronaut, mass $m$

$w = \text{astronaut’s weight} = \frac{Gm_Em}{r^2}$

$r = \text{astronaut’s distance from the center of the earth}$

$r - R_E = \text{astronaut’s distance from the surface of the earth}$
it's interesting to follow the density as one proceeds from crust to mantle to core
Eratosthenes knew that on the summer solstice at local noon in the town of Syene on the Tropic of Cancer, the sun would appear at the zenith, directly overhead. He also knew, from measurement, that in his hometown of Alexandria, the angle of elevation of the Sun would be 1/50 of a full circle (7°12') south of the zenith at the same time. Assuming that Alexandria was due north of Syene he concluded that the distance from Alexandria to Syene must be 1/50 of the total circumference of the Earth. His estimated distance between the cities was 5000 stadia (about 500 miles).

Eratosthenes worked at Alexandria. In 236 BC he was appointed as a librarian of the Alexandrian library. In 194 BC Eratosthenes became blind and a year later he supposedly starved himself to death.
Physics of Tides

Moon pulls on Earth’s tidal bulge to slow Earth’s rotation.

Earth’s tidal bulge pulls Moon ahead in its orbit—Moon spirals outward in its orbit.

Earth’s spin slows down

http://www.astronomynotes.com/gravappl/s10.htm
Gravitational potential energy

- Objects changing their distance from earth are also changing their potential energy with respect to earth.

The gravitational force is conservative: The work done by $\vec{F}_g$ does not depend on the path taken from $r_1$ to $r_2$.

Gravitational potential energy $U = -\frac{Gm_m m}{r}$ for the system of the earth and the astronaut.

$U$ is always negative, but it becomes less negative with increasing radial distance $r$. 
\[ F_g = G \frac{mM}{r^2} \]

\[ W = \int_1^2 \vec{F}_g \cdot d\vec{l} = \int_{r_0}^{r} F_g \, dr = GmM \int_{r_0}^{r} \frac{dr}{r^2} = \]

\[ = -GmM \left( \frac{1}{r} - \frac{1}{r_0} \right) = -\frac{GmM}{r} = U_g \]
Escape velocity

"before"

\[ m \rightarrow v_0 \]
\[ U_0 = -\frac{GM}{R} \]

"after"

\[ m \rightarrow v_f = 0 \]
\[ U_f = 0 \]

\[ \frac{m v_f^2}{2} - \frac{GMmM}{R} = 0 \]

\[ v_f^2 = \sqrt{\frac{2GM}{R}} = \sqrt{2Rg} = \sqrt{2 \times 9.8 \times 6.4 \times 10^6 \frac{m}{s}} = 11 \frac{km}{s} \]

\[ Rg = \frac{GM}{R^2} \; \text{; } 2\pi R = 40,000 \text{ km} \]
\[ R \approx 6.4 \times 10^6 \text{ m} \]
Satellite motion.

\[ F = G \frac{M m}{r^2} = ma = m \frac{v^2}{r}; \quad G \frac{M m}{r^2} = m \frac{v^2}{r} \]

\[ v = \sqrt{\frac{GM}{r}} \]

\[ T = \frac{2\pi r}{v} = \frac{2\pi r \sqrt{F}}{\sqrt{GM}} = \frac{2\pi r^{3/2}}{\sqrt{GM}} \]
Geosynchronous satellite \( G = 6.67 \times 10^{-11} \text{Nm}^2\text{kg}^{-1} \)
\( M = 5.98 \times 10^{24} \text{kg} \)

\[ T = 24 \text{ hours} = 24 \times 60 \text{ min} \times 60 \text{ s} = 86,400 \text{ s} \]

\[ r^{3/2} = \frac{T}{2\pi} \sqrt{GM} ; \quad r = (\frac{T}{2\pi})^{2/3} (GM)^{1/3} \approx 42,000 \text{ km} \]

\[ v = \sqrt{\frac{GM}{r}} \approx 3.1 \text{ km/s} \]

\( \sqrt{\frac{N \text{m}^2}{\text{kg} \cdot \text{kg}}} = \frac{\text{m}}{\text{s}} \)
Another example

\[ T = \frac{2\pi R^{3/2}}{\sqrt{GM}} = \sqrt{\frac{\pi}{3}} \frac{1}{\sqrt{G\rho}} \]

\[ M = \frac{4}{3} \pi R^3 \rho \]

\( = 80 \text{ min} \)
Binary star

\[ F = G \frac{m \cdot m}{(2r)^2} \quad \Rightarrow \quad m a = m \frac{v^2}{r} \]
A brief history of Gravity
Claudius Ptolemy  (85 – 165)
Alexandria, Egypt
Nicolai Copernic
(1473 – 1543) Poland
Johannes Kepler
(1571 – 1630) Germany
Kepler’s Laws
Kepler’s 1st Law

Kepler: The path of each planet about the Sun is an ellipse with the Sun at one focus.

Newton: Gravity gives the centripetal force!
Kepler's 2nd Law

**Kepler**: Each planet moves so that an imaginary line drawn from the Sun to the planet sweeps out an equal area in equal periods of time.

**Newton**: This is explained by angular momentum conservation.
Kepler’s 3\textsuperscript{rd} Law

\[
\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{S_1}{S_2}\right)^3
\]
Galileo Galilei
(1564 – 1642) Italy
Isaac Newton
(1643 – 1727) England
Newton’s law of gravity:

“There is a force of attraction between every particle in the universe, which is proportional to the product of their masses and inversely proportional to the square of the distance between them. This force acts along the line joining the particles.”
Albert Einstein
(1879 – 1955)
Weightlessness

What is the difference between being in “free fall” and being “out of the reach of large gravitational forces?”

\[ w = 0 \]

\[ m g \]

\[ a = g \text{(down)} \]