Spin Hall Effect in 2D disordered systems with spin-orbit coupling

Shuichi Murakami
(Department of Applied Physics, University of Tokyo)

Collaborators: Naoto Nagaosa (Tokyo)
Shoucheng Zhang (Stanford)
Naoyuki Sugimoto (Tokyo)
Shigeki Onoda (Tokyo)
Spin Hall effect (SHE)

Electric field induces a transverse spin current.

- **Extrinsic spin Hall effect**  
  D'yakonov and Perel’ (1971)  

Impurity scattering = spin dependent (skew-scattering)

Spin-orbit coupling

- **Intrinsic spin Hall effect**  
  Berry phase in momentum space

No impurities required!
**Intrinsic spin Hall effect in semiconductors**

\[ j_j^i = \sigma_s \varepsilon_{ijk} E_k \]

- \(i\): spin direction
- \(j\): current direction
- \(k\): electric field

\(\sigma_s\): even under time reversal = reactive response (dissipationless)

- Nonzero in nonmagnetic materials.

Cf. Ohm’s law: \( j = \sigma E \)

\(\sigma\): odd under time reversal = dissipative response
Intrinsic spin Hall effect

- p-type semiconductors  
  Luttinger model

\[ H = \frac{\hbar^2}{2m} \left[ \left( \gamma_1 + \frac{5}{2} \gamma_2 \right) k^2 - 2\gamma_2 (\vec{k} \cdot \vec{S})^2 \right] \]

( \( \vec{S} \) : spin-3/2 matrix)

- 2D n-type semiconductors in heterostructure
  Rashba model

\[ H = \frac{k^2}{2m} + \lambda (\vec{\sigma} \times \vec{k})_z \]
Experiments on spin Hall effect

- **3D n-type, spin accumulation at the edges**

- **2D p-type, spin LED**
Criterion for nonzero spin Hall conductivity

- Different filling for bands in the same multiplet of \( \vec{J} = \vec{L} + \vec{S} \).
  \[ \langle c | H_{\text{s.o.}} | v \rangle \neq 0 \]

(Example) : GaAs

Valence band: \( J=3/2 \)
Hole-doping gives a different filling for HH and LH bands.
\( \rightarrow \text{Spin Hall effect} \)

Conduction band: \( J=1/2 \)
Electron-doping does not give rise to different filling for two conduction bands
\( \rightarrow \text{NO spin Hall effect} \)

In 2D heterostructure, Rashba coupling lifts the degeneracy
\( \rightarrow \text{Spin Hall effect} \)
**Disorder effect, edge effect**

Green’s function method

**Rashba model:**

\[ H = \frac{k^2}{2m} + \lambda(\sigma_x k_y - \sigma_y k_x) \]

+ Intrinsic spin Hall conductivity (Sinova et al. (2003))

\[ \sigma_S = \frac{e}{8\pi} \]

+ Vertex correction in the clean limit (Inoue, Bauer, Molenkamp (2003))

\[ \sigma_{S_{\text{vertex}}} = -\frac{e}{8\pi} \]

\[ \sigma_S = 0 \]

Inoue, Bauer, Molenkamp (2004)

- clean limit
- \( \delta \) -fn. impurity
• **Calculation by Keldysh formalism** (Mishchenko, Shytov, Halperin (2004))
  
  Spin Hall current does not flow at the bulk – consistent with $\sigma_s = 0$

  Spin current only flows near the electrodes
### SHE in disordered Rashba model -- Green’s function--

<table>
<thead>
<tr>
<th></th>
<th>$E_F\tau$</th>
<th>$\Delta\tau$</th>
<th>Potential</th>
</tr>
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<tbody>
<tr>
<td>Inoue, Bauer, Molenkamp</td>
<td>$\infty$</td>
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<td>Mishchenko, Shytov,</td>
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<td>Halperin (2004)</td>
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<td></td>
<td>• Diffusion equation</td>
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<td>• Arbitrary form of Rashba coupling (?)</td>
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\[
J_s = \frac{1}{2} \{ v_y, S_z \} \propto \frac{dS_y}{dt} \quad \rightarrow \quad \langle J_s \rangle = 0 \quad \text{for steady state}
\]
Definition of spin current is not unique in the presence of spin-orbit coupling

- Spin-orbit coupling → spin is not conserved → no unique def. of spin current
- Noether’s theorem cannot be applied.

\[
\frac{\partial S_i}{\partial t} + \nabla \cdot J_i^{\text{(spin)}} = 0 \iff 0 = \frac{\partial}{\partial t} \int S_i d^d r = i[H, \int S_i d^d r]
\]

Eq. of continuity requires conservation of spin, but the spin is not conserved in these models

“Conventional” definition of the spin current:

\[
J_s = \frac{1}{2} \{v_y, S_z\}
\]
SHE in disordered Rashba model -- Green’s function + numerics --

DC spin Hall conductivity is likely to vanish.

\[ \sigma_s \approx 0 \]
Outline

Disorder effect in **Rashba model**

- **Conventional spin current** \( J_s = \frac{1}{2} \{ v_y, S_z \} \)
  - Keldysh formalism
    - \( \sigma_s = 0 \) in the bulk under general conditions

- **Conserved effective spin current**
  (Zhang et al. ('05))
  - Depends on impurity potential: Extrinsic
  - Nonzero but much smaller than \( \frac{e}{8\pi} \)

For general system: \( \sigma_s \) is nonzero.
### SHE in disordered Rashba model -- Green’s function--

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<tr>
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Dimitrova; Chalaev, Loss: \[ J_s = \frac{1}{2} \{ v_s, S_z \} \propto \frac{dS_y}{dt} \rightarrow \langle J_s \rangle = 0 \] for steady state
\[ \langle J_s \rangle = \frac{1}{2i} \int \frac{d\omega}{2\pi} \int \frac{d^2 p}{(2\pi)^2} \text{tr}(J_s G^K) \]

When \( J_s \) is proportional to \( \dot{S}_y \) ... (e.g.) Rashba model: \( H = \frac{k^2}{2m} + \lambda (\vec{\sigma} \times \vec{k})_z \)

\[ J_s \equiv \frac{1}{2} \{\nu_y, S_z\} = \frac{1}{4im\lambda} [H, \sigma_y] \]

\[ \langle J_s \rangle = \frac{1}{2i} \frac{1}{4im\lambda} \int \frac{d\omega}{2\pi} \int \frac{d^2 p}{(2\pi)^2} \text{tr}(\sigma_y [H, G^K]) \]

\[
[H, G^K] = -ie\vec{E} \cdot \partial_{\vec{p}} G^K - \frac{i}{2} e\vec{E} \cdot \{\partial_{\vec{p}} H, \partial_\omega G^K\}
- \frac{i}{2} e\vec{E} \cdot (\partial_{\vec{p}} \Sigma \partial_\omega G - \partial_\omega \Sigma \partial_{\vec{p}} G)_K
+ \frac{i}{2} e\vec{E} \cdot (\partial_{\vec{p}} G \partial_\omega \Sigma - \partial_\omega G \partial_{\vec{p}} \Sigma)_K - ([\Sigma, G])_K
\]

No contribution to \( \langle J_s \rangle \)
If $\hat{O}$ is independent of $\vec{r}$ and $t$, $\left\langle \frac{d\hat{O}(t)}{dt} \right\rangle = 0$ in the steady state under uniform $\vec{E}$.

- Rashba model
  (conventional) spin current operator $J_s = \frac{1}{2} \{v_y, S_z\} \propto \frac{dS_y}{dt}$ $\Rightarrow \left\langle J_s \right\rangle = 0$

- Charge current in diffusive metal $\vec{J} \equiv e \frac{d\vec{r}}{dt}$ $\Rightarrow \left\langle \vec{J} \right\rangle \neq 0$
Conserved spin current

\[ J_s \equiv \frac{d}{dt} (yS_z) \]

Eq. of continuity for spin

\[ \nabla \cdot J_s + \frac{d}{dt} S_z = 0 \]

Calculation of \( \langle J_s \rangle \) in response to electric field?

1) Spatial modulation of \( \vec{E} \) : \( \vec{E} = (E e^{iqy}, 0, 0) \)

2) Calculate \( \langle \dot{S}_z \rangle = \frac{1}{2i} \int \frac{d\omega}{2\pi} \int \frac{d^2 p}{(2\pi)^2} \text{tr} (\dot{S}_z G^K) \)

3) Calculate \( \langle J_s \rangle \) by taking \( \partial_q \)

\[ \langle J_s \rangle = \frac{1}{2i} \lim_{q \to 0} (-i\partial_q) \int \frac{d\omega}{2\pi} \int \frac{d^2 p}{(2\pi)^2} \text{tr} (\dot{S}_z G^K) \]
\[ \langle j_s \rangle = \frac{1}{2i} \lim_{q \to 0} \frac{1}{iq} \int \frac{d\omega}{2\pi} \int \frac{d^2 p}{(2\pi)^2} \text{tr}(S_z G^K) \rightarrow i \text{tr}(H, S_z G^K) = -i \text{tr}(S_z [H, G^K]) \]

- **No contribution**

\[
[H, G^K] = -ieE e^{iqy} \partial_{p_x} G^K - \frac{i}{2} eE e^{iqy} \left\{ \partial_{p_x} H, \partial_\omega G^K \right\} - \frac{q}{2} \left\{ \partial_{p_x} H, G_E^K \right\} \]

- **No contribution for**
  - 1st Born
  - higher Born
  - weak localization corr.

- **Conventional spin current** \( \langle J_s \rangle \)

\[
\frac{-1}{2} eE t q e^{iqy} \varepsilon_{ijz} \left\{ \partial_{p_i} H, \partial_{p_j} G^K \right\}
\]

\[
\frac{-q}{2} eE \left( \partial_{p_i} \Sigma \partial_{p_j} G - \partial_{p_i} G \partial_{p_j} \Sigma \right)_K
\]

\[
\frac{-i}{2} \left( \tilde{\partial}_t \Sigma \partial_\omega G - \partial_\omega \Sigma \tilde{\partial}_t G - \tilde{\partial}_t G \partial_\omega \Sigma + \partial_\omega G \tilde{\partial}_t \Sigma \right)_K
\]

- **Torque dipole density** \( \langle P_s \rangle \)

\[
- \frac{q}{2} \left( \frac{\partial \Sigma^R}{\partial p_y} G_E^K + G_E^K \frac{\partial \Sigma^A}{\partial p_y} - \Sigma_E^K \frac{\partial G^A}{\partial p_y} - \frac{\partial G^K}{\partial p_y} \Sigma_E^K \right) - (\left[ \Sigma, G \right])_K
\]
\[
\langle P_s \rangle = \frac{1}{4i} \int \frac{d\omega}{2\pi} \int \frac{d^2 p}{(2\pi)^2} \text{tr} \left( S_z \left( \frac{\partial \Sigma}{\partial p_y} G + G \frac{\partial \Sigma}{\partial p_y} \right) \right)_{E,K}
\]

\[
\langle J_s \rangle = \frac{1}{4i} \int \frac{d\omega}{2\pi} \int \frac{d^2 p}{(2\pi)^2} \text{tr} \left( S_z \left( \frac{\partial H}{\partial p_y} G + G \frac{\partial H}{\partial p_y} \right) \right)_{E,K}
\]

\[
= \frac{1}{2i} \int \frac{d\omega}{2\pi} \int \frac{d^2 p}{(2\pi)^2} \text{tr} \left( \frac{1}{2} \left\{ \frac{\partial H}{\partial p_y}, S_z \right\} G_K \right)
\]

\[
J_s \equiv \frac{1}{2} \{v_y, S_z\}
\]
\[
\langle P_s \rangle = \frac{1}{4i} \int \frac{d\omega}{2\pi} \int \frac{d^2 p}{(2\pi)^2} \text{tr} \left( S_z \left( \frac{\partial \Sigma}{\partial p_y} G + G \frac{\partial \Sigma}{\partial p_y} \right)_E \right)
\]

In general systems
- No divergence in the clean limit
- \( \propto \frac{\partial V}{\partial \vec{p}} \rightarrow \langle P_s \rangle \) always extrinsic zero for \( \delta \)-fn. impurity

\( \langle J_s \rangle_{\text{1st}} = 0 \)

\( \langle J_s \rangle_{\text{2nd}} \neq 0 \)

Rashba model

\( \alpha \): range of impurity pot. (short but finite)

- \( \sigma_s = 0 \) in the clean limit
- \( \sigma_s = 0 \) in the short-range limit

Typically \( \frac{\lambda p_F}{E_F} < 0.2, \frac{1}{\tau E_F} < 0.05 \) \( \rightarrow \sigma_s \ll 10^{-5} \frac{e}{8\pi} \)
### Spin Hall effect in the Rashba model

<table>
<thead>
<tr>
<th>Impurity potential</th>
<th>Born approx.</th>
<th>Conventional spin current $J_s$</th>
<th>Torque dipole density $P_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta(\vec{r})$</td>
<td>1\textsuperscript{st}</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>higher</td>
<td>0</td>
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<td>$V(\vec{p} - \vec{p'})$</td>
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</table>

Zero in general -- $P_s$ is extrinsic.

Nonzero for general spin-orbit-coupled system
On Rashba-like tight-binding models ... 

This case also satisfies \( J_x \propto \hat{S}_y \) \( \langle J_x \rangle = 0 \) No intrinsic SHE

intrinsic SHE \( \sigma_s \approx \frac{e}{8\pi} \) (const.)

\[ \begin{align*}
-\nu_i + i\nu_2 \sigma_z & \quad -\nu_i - i\nu_2 \sigma_z \\
-\nu_i + i\nu_2 \sigma_z & \quad -\nu_i - i\nu_2 \sigma_z
\end{align*} \]

"Ando model"

(a) \( \nu_2/\nu_1 = 0.2, \nu_3/\nu_1 = 0 \)

without vertex correction

with vertex correction

\[ \sigma_{yx} \left( \times \frac{e}{8\pi} \right) \]

\( N(\varepsilon) \)

(b) \( \nu_2/\nu_1 = 0.2, \nu_3/\nu_1 = 0.1 \)
In Rashba-like systems
• Intrinsic SHE (if nonzero) is of the order of $\frac{e}{8\pi}$.
Summary

- Rashba model

conserved effective spin current \( J_s = J_s + P_s \)

Conventional spin current \( J_s = \frac{1}{2} \{ v_y, S_z \} \)

\( J_s \propto \dot{S}_y \) yields \( \langle J_s \rangle = 0 \) under general conditions.

- Any impurity potential
- Finite \( \tau \)
- Higher-order Born approx. + weak localization corr.

Torque dipole density \( P_s \)

Nonzero but depends on the impurity pot. -- extrinsic
Zero in the clean limit
Zero for \( \delta \)-fn. potential
Small for Rashba model: \( \sigma_s \ll 10^{-5} \cdot \frac{e}{8\pi} \)

For generalized models it gives \( \sigma_s \approx O\left(\frac{e}{8\pi}\right) \)
Nonzero spin Hall effect in band insulators

1) Zero-gap semiconductors: $\alpha$-Sn, HgSe, HgTe, $\beta$-HgS...

- Spin Hall effect is nonzero ($\approx 0.1 e/a$) in band insulators
- Uniaxial strain $\rightarrow$ finite gap at $k=0$

Spin Hall insulator
2) Narrow-gap semiconductors: PbS, PbSe, PbTe

Direct gap (0.15eV-0.3eV) at 4 equivalent L-points

\[ H = \mathbf{\nu} \mathbf{k} \cdot \mathbf{\hat{p}} \tau_1 + \lambda \mathbf{\nu} \mathbf{k} \cdot (\mathbf{\hat{p}} \times \mathbf{\sigma}) \tau_2 + M \mathbf{\nu}^2 \tau_3 \]

\( \mathbf{\sigma} \): spin \hspace{1cm} \( \mathbf{\tau} \): orbital

- Spin Hall effect is nonzero (\( \approx 0.04e/a \)) in band insulators

**Spin Hall insulator**