Spin-Hall conductivity due to Rashba spin-orbit interaction in disordered systems

Oleg Chalaev and Daniel Loss

Department of Physics and Astronomy, University of Basel, Klingelbergstrasse 82, Basel, CH-4056, Switzerland

8th August 2005

Outline

• Spin-Hall effect in pure material
• Kubo-Greenwood formula
• Zero-loop approximation
• Weak localization correction
• Spin current and the magnetization
• Conclusions
Spin-Hall effect

J. Sinova et al. 2004: The case of no impurities

$$\sigma_{yx}^z = \frac{e}{8\pi}$$

J. Inoue et al., 2004: In the diffusive case, due to the vertex renormalization,

$$\sigma_{yx}^z = 0$$

Other papers confirmed this result: Mishchenko et al., Khaetskii, Raimondi & Schwab, Dimitrova.

Our scope: improve the precision of the calculation.
General relations

The Hamiltonian:

\[ \hat{H} = \frac{\hat{p}^2}{2m} + \alpha(\hat{\sigma}^1\hat{p}_y - \hat{\sigma}^2\hat{p}_x) + U(\vec{r}), \]

where \( U(\vec{r}) \) is a disorder potential,

\[ \langle U(\vec{r})U(\vec{r}')\rangle = \frac{1}{2\pi\nu\tau}\delta(\vec{r} - \vec{r}'). \]

\( \alpha(\hat{\sigma}^1\hat{p}_y - \hat{\sigma}^2\hat{p}_x) \) is a Rashba SO term,

which leads to a modification of the charge current operator:

\[ \hat{p} \rightarrow \hat{p} - \frac{e}{c}\vec{A}, \]

\[ \vec{A} = - (\alpha mc/e)(-\hat{\sigma}^2, \hat{\sigma}^1, 0) = \text{effective vector potential.} \]
General relations

The Hamiltonian:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \alpha (\hat{\sigma}^1 \hat{p}_y - \hat{\sigma}^2 \hat{p}_x) + U(\vec{r}),$$

Evaluate spin current assuming:

- Linear response in electric field $\implies$ Kubo formula.
- We use the disorder averaging diagrammatic technique, $p_F l \gg 1$.
- Rashba SOI $\alpha p_F \ll E_F$
A generalized Kubo-Greenwood formula

\[ \sigma^z_{yx} = \frac{e}{2\pi m^2} \text{Tr} \left[ \frac{\sigma^3}{2} p_y \hat{G}_R \left( p_x - \frac{e}{c} \hat{A}_x \right) \hat{G}_A \right] \]

Loop expansion:

\[ \sigma^z_{yx} = |e| \sum_n \frac{s_n}{(p_F l)^n}, \quad p_F l \gg 1, \]

\( l \) = mean scattering free path

The charge current operator

\[ \hat{p}_x - \frac{e}{c} \hat{A}_x = p_F \hat{n}_x + \left( \hat{p}_x - p_F \hat{n}_x - \frac{e}{c} \hat{A}_x \right), \quad \hat{n}_x \equiv \frac{\hat{p}_x}{p} \]
A generalized Kubo-Greenwood formula

\[ \sigma_{yx}^z = \frac{e}{2\pi m^2} \text{Tr} \left[ \frac{\sigma^3}{2} p_y \hat{G}_R \left( p_x - \frac{e}{c} \tilde{A}_x \right) \hat{G}_A \right] \]

Loop expansion:

\[ \sigma_{yx}^z = |e| \sum_n \frac{s_n}{(p_F l)^n}, \quad p_F l \gg 1, \]

\( l = \text{mean scattering free path} \)

The charge current operator

\[ \hat{p}_x - \frac{e}{c} \tilde{A}_x = p_F \hat{n}_x + \left( \hat{p}_x - p_F \hat{n}_x - \frac{e}{c} \tilde{A}_x \right), \quad \hat{n}_x \equiv \frac{\hat{p}_x}{p} \]

- In zero loop diagrams, gives the contribution \( \propto e(p_F l) \) to \( \sigma_{yx}^z \)
- In first loop diagrams, gives the contribution \( \propto e \) to \( \sigma_{yx}^z \)
A generalized Kubo-Greenwood formula

\begin{equation*}
\sigma_{yx}^z = \frac{e}{2\pi m^2} \text{Tr} \left[ \frac{\sigma^3}{2} p_y \hat{G}_R \left( p_x - \frac{e}{c} \tilde{A}_x \right) \hat{G}_A \right]
\end{equation*}

Loop expansion:

\begin{equation*}
\sigma_{yx}^z = |e| \sum_n \frac{s_n}{(p_F l)^n}, \quad p_F l \gg 1,
\end{equation*}

\(l = \text{mean scattering free path}\)

The charge current operator

\begin{equation*}
\hat{p}_x - \frac{e}{c} \tilde{A}_x = p_F \hat{n}_x + \left( \hat{p}_x - p_F \hat{n}_x - \frac{e}{c} \tilde{A}_x \right), \quad \hat{n}_x \equiv \frac{\hat{p}_x}{p}
\end{equation*}

- In zero loop diagrams, gives the contribution \(\propto e\) to \(\sigma_{yx}^z\)
- In first loop diagrams, gives the contribution \(\propto \frac{e}{p_F l}\) to \(\sigma_{yx}^z\)
Thus, taking into account the contribution from

\[
\left( \hat{p}_x - p_F \hat{n}_x - \frac{e}{c} \tilde{A}_x \right)
\]

in zero-loop diagrams means we have to consider the contribution from

\[ p_F \hat{n}_x \]

in first-loop diagrams.
Zero-loop approximation

Boltzmann diagram

vertex correction

spin current vertex

charge current vertex

spin current vertex

diffusion

charge current vertex
Zero-loop approximation

Boltzmann diagram

vertex correction

+ 

spin current vertex

diffusion

charge current vertex
Zero-loop approximation

Boltzmann diagram

\[ \left( \hat{p}_x - \frac{e}{c} \tilde{A}_x \right) + \left( \hat{p}_x - \frac{e}{c} \tilde{A}_x \right) \]

vertex correction

spin current vertex

spin current vertex
diffusion

spin current vertex
Zero-loop approximation

Boltzmann diagram

\( \hat{p}_x - \frac{e}{c} \tilde{A}_x \) + \( \hat{p}_x - \frac{e}{c} \tilde{A}_x \)

\( \hat{p}_x = 0 \)

The renormalization of the charge current vertex results in the **cancellation** of the anomalous term \( \frac{e}{c} \tilde{A} \) in the current vertex (**\( \omega = 0 \)**)
Weak localization correction

about Cooperon and diffusion...
Weak localization correction

\[ e \frac{2\pi m^2}{2\pi m^2} \sum_{\gamma\gamma'=0} \text{Tr} \left\{ \int \frac{d^2 p}{(2\pi)^2} G_A(\vec{p}) p_y \frac{\sigma^3}{2} G_R(\vec{p}) \sigma^\gamma \times \right. \]

\[ \left[ \sigma^\gamma' G_R(\vec{q} - \vec{p}) \left( -p_x - \frac{e}{c} \tilde{A}_x \right) G_A(-\vec{p}) \right]^T \left\} \int \frac{d^2 q}{(2\pi)^2} C^{\gamma\gamma'}(\vec{q}) \right. \]

about Cooperon and diffusion...
Weak localization correction

Click here for the expression.

about Cooperon and diffusion...
Weak localization correction

+ spin vertex correction:

Note: the charge-current vertex renormalization is irrelevant here
Weak localization correction

All together, the diagrams add up to zero

about Cooperon and diffusion...
Connection between the spin current and the magnetization

Without magnetic field and for non-magnetic impurities

\[ \dot{s}_k(t) = -2m\alpha\hat{j}^{s_z}_k(t), \quad k = x, y \]

- valid for arbitrary \( \vec{E} \) and for systems with interaction.

The magnetization

\[ \langle \dot{s}_k \rangle(t) = -2m\alpha\langle \hat{j}^{s_z}_k \rangle(t) \]

In diffusive systems, stationary state is reached at \( t \rightarrow \infty \).
\[ \implies \text{the spin-Hall current must be zero at } \omega = 0. \]
Thus, the result of J. Sinova et al. 2004 for a clean sample

\[ \sigma_{yx}^z = \frac{e}{8\pi} \]

means that

\[ \langle \hat{S} \rangle_y \rightarrow \infty, \quad t \rightarrow \infty. \]
Generalization for the Dresselhaus term

\[ \hat{H}'(\hat{p}, \vec{r}') = \frac{\hat{p}^2}{2m} + U(\vec{r}') + \alpha (\hat{\sigma}_1 \hat{p}_y - \hat{\sigma}_2 \hat{p}_x) + \]
\[ + \beta (\hat{\sigma}_1 \hat{p}_x - \hat{\sigma}_2 \hat{p}_y) + e \vec{r}' \vec{E}, \]

\[ -\frac{1}{2m} \dot{\hat{s}}_x(t) = \alpha \dot{\hat{j}}^{sz}_x(t) + \beta \dot{\hat{j}}^{sz}_y(t), \]
\[ -\frac{1}{2m} \dot{\hat{s}}_y(t) = \alpha \dot{\hat{j}}^{sz}_y(t) + \beta \dot{\hat{j}}^{sz}_x(t), \]

\[ \dot{\hat{j}}_x = -\frac{1}{2m} \frac{\alpha \dot{\hat{s}}_x - \beta \dot{\hat{s}}_y}{\alpha^2 - \beta^2}, \quad \dot{\hat{j}}_y = -\frac{1}{2m} \frac{\beta \dot{\hat{s}}_x - \alpha \dot{\hat{s}}_y}{\beta^2 - \alpha^2}, \quad \alpha \neq \beta. \]

\[ \lim_{t \to \infty} \langle \hat{\hat{j}}^{sz}(t) \rangle = 0 \iff \lim_{t \to \infty} \langle \hat{\hat{s}}_k \rangle(t) = \text{const.}, \quad k = x, y. \]
Conclusions

• We have calculated the zero-loop and the weak localization contributions to the $\sigma^z_{yx}$.

• Both contributions result zero independently.

• General argument: spin-Hall current is zero at $\omega = 0$ and for $B = 0$.

Thanks to: Evgenii Mishchenko & Andrei Shytov.

this document is available on http://shalaev.pochta.ru and here.
References

Disorder averaging diagrammatic technique:
A. A. Abrikosov, L. P. Gor’kov and I. E. Dzyaloshinskii, Methods of quantum field theory in statistical physics, Dobrosvet (Moscow), 1998.

СМ. DVD №5
\[
X_D^{\alpha\beta}(q) = \frac{1}{4\pi\nu\tau} \int \frac{d^2p}{(2\pi)^2} \text{Tr}[\sigma^\alpha G_R(\vec{p}) \sigma^\beta G_A(\vec{p} - q)],
\]

\[
X_C^{\alpha\beta}(q) = \frac{1}{4\pi\nu\tau} \int \frac{d^2p}{(2\pi)^2} \text{Tr}[\sigma^\alpha G_R(\vec{p}) \sigma^\beta G_A^T(q - \vec{p})],
\]

\[
D^{\alpha\alpha} = \frac{1}{4\pi\nu\tau} \frac{1}{1 - X_D^{\alpha\alpha}}, \quad C^{\alpha\alpha'}(q) = \frac{1}{4\pi\nu\tau} \left[ \frac{X_C(q)}{1 - X_C(q)} \right]_{\alpha\alpha'}
\]

See my unofficial notes for more information.
The case of $\alpha = \beta$

\[
\hat{H}_R(p', r') \equiv \hat{H}'(R_\pi/4 \hat{p}, R_\pi/4 \hat{r}) = \frac{\hat{p}'^2}{2m} - 2\alpha \hat{\sigma}^2 \hat{p}_x + U'(r') + e\hat{r}' \vec{E}' \equiv \hat{H}_{R0} + e\hat{r}' \vec{E}',
\]

\[
\hat{\sigma}^{12'} = \frac{1}{\sqrt{2}}(\hat{\sigma}^2 \pm \hat{\sigma}^1), \quad \hat{\sigma}'_3 \equiv \hat{\sigma}^3.
\]

\[
\hat{\rho}(t < 0) = \hat{\rho}_0 = e^{-\hat{H}_{R0}/T} / Z, \quad \langle \hat{\vec{j}}^{s_z} \rangle(t = 0) = 0.
\]

\[
\langle \hat{\vec{j}}^{s_z} \rangle(t) = \text{Tr} \begin{bmatrix} \hat{\sigma}^3 \frac{\hat{p}}{2m} e^{-i\hat{H}_R t} \hat{\rho}_0 e^{i\hat{H}_R t} \end{bmatrix}.
\]

\[
\text{Tr} \begin{bmatrix} \hat{\sigma}^k \hat{H}_R \end{bmatrix}_{\text{spin}} = 0, \quad \text{Tr} \begin{bmatrix} \hat{\sigma}^k \hat{H}_{R0} \end{bmatrix}_{\text{spin}} = 0, \quad k = 1, 3,
\]

\[
\Rightarrow \text{Tr} \begin{bmatrix} \hat{\sigma}^3 \frac{\hat{p}}{2m} e^{-i\hat{H}_R t} \hat{\rho}_0 e^{i\hat{H}_R t} \end{bmatrix} = 0, \quad \forall t.
\]
The period of oscillation is $1/\Delta$, and the exponential decay time is $\tau$. 
Expression for one of the weak localization diagrams

\[ e \frac{1}{2\pi m^2} \sum_{\gamma=0}^{3} \int \frac{d^2 q}{(2\pi)^2} C^{\gamma\gamma}(\vec{q}) \times \frac{1}{2m\tau} \sum_{\mu=0}^{3} A^{\gamma\mu} B^{\mu\gamma}, \]

\[ A^{\gamma\mu} = \text{Tr} \left\{ \int \frac{d^2 p}{(2\pi)^2} G^< (\vec{p}) \sigma^\gamma G^T_A (\vec{-p}) \sigma^\mu \right\}, \]

\[ B^{\mu\gamma} = \text{Tr} \left\{ \int \frac{d^2 p'}{(2\pi)^2} \sigma^\gamma G^> (\vec{-p}') \left[ G_A (\vec{p}') \sigma^\mu \right]^T \right\}, \]

\[ G^< (\vec{p}) = G_A (\vec{p}) p_y \frac{\sigma^3}{2} G_R (\vec{p}), \]

\[ G^> (\vec{-p}) = G_R (\vec{-p}) \left( -p_x - \frac{e}{c} \vec{A}_x \right) G_A (\vec{-p}). \]
Mostly we use colors tgi, blue, OliveGreen, red, green.

The colours from “brown” family are used only in the on-line version, to highlight hyperlinks: Brown, RawSienna, Sepia.

Billissimo likes: Fuchsia, RoyalBlue, OrangeRed