Spin Hall Conductivity in Diffusive Regime

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Rashbe -2DEG:
Spin Hall conductivity $\to 0$ in the diffusive regime.

$\sigma_{xy}^{\sigma_z} \to 0$  \textit{Inoue-Bauer-Molenkamp PRB 70(2004)}

- Linear response theory for diffusive transport
  self-energy $\to$ Drude conductivity
  vertex corrections
- Examples for importance of VC
- Rashba 2DEG
  longitudinal conductivity, spin accumulation
  spin Hall conductivity for non-magnetic impurity
  magnetic impurity
- Summary
Linear Response Theory for Diffusive Conductivity

**Kubo-Greenwood formula – one electron theory**

\[
\sigma_{\mu\nu} = 2\pi\hbar \text{Tr}\langle J_\mu \delta(\varepsilon_F - H)J_\nu \delta(\varepsilon_F - H)\rangle_{av}
\]

\[
\rightarrow \frac{\hbar}{\pi} \text{Tr}\langle J_\mu G^R J_\nu G^A \rangle_{av}
\]

\[
\langle \ldots \rangle_{av} : \text{random average}
\]

**Self energy**

\[
\langle G^{R(A)}(\omega) \rangle_{av} \equiv \tilde{G}^{R(A)}(\omega) = \frac{1}{\omega + (-)i0 - \varepsilon_k - \Sigma^{R(A)}(\omega)}
\]

Born approx., SC-Born, CPA, etc.
Renormalization of GF by *summation over infinite series of expansion*

**Drude conductivity**

\[
\sigma_{xx} \rightarrow \frac{\hbar}{\pi} \text{Tr}_x \tilde{G}^R J_x \tilde{G}^A \frac{\hbar e^2}{\pi} \sum_k k_x^2 \tilde{G}^R \tilde{G}^A \frac{\hbar e^2}{\pi} \sum_k k_x^2 \frac{2\pi\tau}{\hbar} \delta(\varepsilon_F - \varepsilon_k) = \frac{ne^2\tau}{m}
\]

Boltzmann equation, *second order perturbation*

\[
\rho = \frac{1}{\sigma}
\]
Vertex Corrections (VC)

**Summation over infinite series of expansion**

\[
\langle G^R J_y G^A \rangle_{av} = \langle G^A \rangle_{av} J_y \langle G^R \rangle_{av} + \text{corrections}
\]

\[
= \langle G^A \rangle_{av} \tilde{J}_y \langle G^R \rangle_{av}
\]

\[
\tilde{J}_\mu = J_\mu + VC
\]

- **Distinction between energy relaxation time (\(\Sigma\)) and momentum relaxation time (1-cos\(\theta\))**: forward scattering does not contribute the resistivity
- **Isotropic scattering potentials**: VC \(\rightarrow 0\) as \(J_x \sim k_x\)
- **VC is non-zero when the system is anisotropic even for isotropic potentials**
- **Ward identity should be satisfied** (conservation theorem)

**Examples**

CPP-GMR
TMR – VC to conductivity dominates

*H. Itoh, JI et al, JPSJ (1999)*

*H. Itoh, T. Ohsawa, JI, PRL(2000)*
Rashba SO - 2DEG

\[ H_0 = \begin{pmatrix} \frac{\hbar^2 k_x^2}{2m} & i\langle \alpha E_z \rangle k_- \\ -i\langle \alpha E_z \rangle k_+ & \frac{\hbar^2 k_y^2}{2m} \end{pmatrix} \]

\[ \langle \alpha E_z \rangle / \hbar \equiv \lambda \]

Questions
1. Does the effective magnetic field make the conductivity spin-dep.? → no
2. Does the spin accumulation exist? → yes
3. How is the Spin-Hall conductivity in diffusive regime?  
   JI et al. PRB (2003)
Our approach

**Intrinsic SO interaction + effect of scattering**

**Kubo formula**
with eigen-states of Rashba-2DEG Hamiltonian, 
Born approx. + vertex corrections
Isotropic random potentials

**Results**

\[
\sigma_{xx}^{\uparrow\uparrow} = \sigma_{xx}^{\uparrow\downarrow} = \frac{e^2 n_0 \tau}{m} + e^2 \tau D \lambda^2
\]

\[
\sigma_{xx}^{\uparrow\downarrow} = 0
\]

\[
\langle s_y \rangle = 4 \pi e \tau D \lambda E
\]

SO interaction
In-plane spin accumulation
Numerical simulation on longitudinal conductivity

Finite size $W=100 - 200$ sites

$L < 30000$ sites

Analytical result (Inoue et al. PRB67) $\sigma = \sigma_D + e^2 D \tau \lambda_{SO}^2$

Localization length

$\Rightarrow$ Cross-over of the universality class

$\xi = 2800a$ for $\lambda_{SO} = 0.0$

$\xi = 10600a$ for $\lambda_{SO} = 0.1t$

Sample width should be several times larger than the mean free path (50a – 60a).

H. Itoh et al. Physica E in press

Finite size $W=100 - 200$ sites

$L < 30000$ sites
Effect of disorder on spin-Hall conductivity

Current operators,

\[ J_{x(y)} = e \left[ \frac{\hbar}{m} k_{x(y)} \mathbf{1} - (\pm) \lambda \hbar \sigma_y x(x) \right] \]

Spin current operator

\[ J_y^z = \frac{\hbar}{4} \{ \nu_y, \sigma_z \} = \frac{\hbar^2}{2m} k_y \sigma_z \]

Spin Hall conductivity

\[ \sigma^z_{yx}(\omega) = \frac{e}{8\pi} \frac{\lambda + \lambda'(\omega)}{\lambda} \]

\[ \lambda'(\omega) = -\frac{\lambda}{1 - i\tau\omega} \]

Isotropic & weak scattering limits

Self-energy broadening < SO interaction

Interpretation

- SHE in diffusive regime depends on the type of SO interaction


2DHG: SO interaction \( \sim k^3 \)

\[ J_x^{SO} = 6\beta k_x k_y \sigma_x + 3\beta \left( k_y^2 - k_x^2 \right) \sigma_y \]
Effect of disorder on spin-Hall conductivity

Interpretation - continued

Scattering $k \rightarrow k'$

$\Rightarrow$ Scrambling the spin precession

$\Rightarrow$ Depend on SO interaction

Adagidel-Bauer (2005)

Does SH current always vanishes in Rashba 2DEG?

• Zeeman term makes SH current finite. Adagidel-Bauer (2005)

• Rashba’s general argument excludes the Zeeman term.

Spin dependent impurity potentials?
### Diffusive transport regime in Rashba-2DEG

**Magnetic impurity – spin-flip scattering**  (Elliott-Yafet)

\[
V_m(\mathbf{r}) = J \sum_i s \cdot S_\delta(\mathbf{r} - \mathbf{R}_i) \\
= J \sum_i \left[ \frac{1}{2} (\sigma_+ S_- + \sigma_- S_+) + \sigma_z S_z \right] \delta(\mathbf{r} - \mathbf{R}_i)
\]

\[
\Sigma(\omega) = \frac{J^2 \langle S^2 \rangle}{2L^2} n \sum_{ks} g_{ks}(\omega) \approx \pm i \frac{\hbar}{2} \left( \frac{1}{\tau} + \frac{1}{\tau_{sf}} \right) \approx \pm i \frac{\hbar}{2} \frac{3}{2\tau}
\]

<table>
<thead>
<tr>
<th></th>
<th>ballistic</th>
<th>para – imp</th>
<th>mag – imp</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_{xx})</td>
<td>(\sigma_D + 2e^2 D_0 \tau \lambda^2)</td>
<td>(\sigma_D + 2e^2 D_0 \tau \lambda^2 + \lambda (\lambda + \lambda') e^2 D_0 \tau \Delta)</td>
<td></td>
</tr>
<tr>
<td>(\sigma_{xy})</td>
<td>(\frac{e}{8\pi})</td>
<td>0</td>
<td>(\frac{e(\lambda + \lambda')}{8\pi \lambda})</td>
</tr>
<tr>
<td>(\langle S_y \rangle)</td>
<td>(-4\pi e D_0 \tau \lambda E)</td>
<td>(4\pi e D_0 \tau [\lambda - (\lambda + \lambda')(2 + \Delta)] E)</td>
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\(\lambda' = -\lambda \frac{\Delta}{8 + \Delta}\)  
Non-magnetic impurity  
\(\Delta \square 16\pi^2 \lambda^2 n_0\)  
\(1/\tau_{sf} \square 2/\tau\)

**Spin Hall current is non-zero even in diffusive regime for 2DEG.**
Summary

• A brief review on Kubo formula and vertex corrections
• Rashba split 2DEG, diffusive regime
  Born approximation and Ladder diagrams

- No spin dependence in conductivity
- Extra term in longitudinal conductivity due to spin-orbit interaction
- Spin accumulation exists
- Suppression of Spin Hall current by non-magnetic disorder
- Spin Hall current is non-zero even in diffusive regime for magnetic impurities
  ➤ Experimental confirmation