Mesoscopic Spin Hall Effect in Pictures: Total Spin Currents, Local Spin Fluxes, and Flowing Spin Densities

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References: PRL 94, 106602 (2005); PRL 95, 046601 (2005); cond-mat/0408693 (PRB); cond-mat/0503415 (PRB); cond-mat/0506588.
Breakthrough Experiments: Nonequilibrium Spin Hall Accumulation

- Optical detection of lateral spin accumulation in thin films of GaAs:
- Optical detection of lateral spin accumulation in 2D hole gas:


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Charge vs. Spin Hall Effect
“Old” Theory: **Extrinsic** Spin Hall Effect in Paramagnetic Systems with SO Impurities

- D’yakonov and Perel’ (1971), Hirsch (1999), Zhang (2000): **SO dependent** scattering off impurities $\lambda \hat{\sigma} \cdot [\hat{p} \times \nabla V(\mathbf{r})]$

- **Recent revival**: Engel, Halperin, and Rashba (2005); Tse and Das Sarma (2005) - **skew-scattering** + side jumps
“New” Theory: Intrinsc Spin Hall Effect in Semiconductors with Spin-Split Bloch Bands

Murakami, Nagaosa, and Zhang, Science 301, 1348 (2003):

\[ \frac{dr}{dt} = \frac{1}{\hbar} \frac{d\varepsilon_n(k)}{dk} + \frac{dk}{dt} \times B_n(k) \]

\[ B_n(k) = \nabla \times A_{nk}, \quad A_{nk} = -i\langle nk | \frac{\partial}{\partial k_\alpha} | nk \rangle \]

\[ j = -e \int \frac{d^d k}{(2\pi)^d} \sum_n f_n(k) v_n(k) \Rightarrow \sigma_{SH} = \frac{e}{4\pi^2} \frac{\gamma_1 + 2\gamma_2}{\gamma_2} (k_F^x - k_F^y) \]


\[ \sigma_H = \frac{j_y}{E_x} \]

\[ \frac{e}{8\pi} \]

\[ 0? \]

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Intrinsic and Extrinsic “Controversies”

- What is the proper definition of spin currents in a medium with SO coupling and can they be measured?
  
  \[
  \frac{d\hat{S}^k}{dt} = \frac{1}{i\hbar} [\hat{S}^k, \hat{H}] \Leftrightarrow \frac{d\hat{S}^k}{dt} + \nabla \cdot \hat{j}^k = \hat{F}^k
  \]
  
  \[
  \hat{j}^k_i = \frac{\hbar}{4} (\hat{\sigma}_k \hat{\nu}_i + \hat{\nu}_i \hat{\sigma}_k)
  \]

- Can we estimate the extrinsic spin Hall current magnitude without invoking any “intrinsic” SO coupling related huge enhancement factors?

  \[
  \sigma_{sH}^{\text{extr}} \sim \frac{e^2}{\hbar} (\lambda_c k_F)^2 k_F
  \]

  \[
  (\lambda_c k_F)^2 = \left( \frac{\hbar k_F}{mc} \right)^2 \sim 10^{-7}
  \]

- Do intrinsic spin Hall currents, carried by the whole Fermi sea, really transport spins between two points in space, thereby generating spin accumulation and offering semiconductor-based spin injector?

- How does the intrinsic spin Hall current behaves in the presence of disorder (spin-independent scattering off static impurities)?

- Can we control the extrinsic spin Hall current just by playing with gate electrodes?
Rashba Spin-Orbit Coupling in 2DEG

Inversion symmetry $\Rightarrow$ no R-SO

Broken inversion symmetry $\Rightarrow$ R-SO

\[
\hat{H} = \frac{\hat{p}^2}{2m} + \frac{\alpha}{\hbar} (\hat{\sigma} \times \hat{p})_z
\]

$\langle \alpha E_z \rangle$ is the expectation value over the lowest subband with energy $E_0$
Can Nonequilibrium Spin Hall Accumulation Be Induced in Ballistic Semiconductor Nanostructures?

\[
\langle S(r) \rangle = \frac{\hbar}{2} \int_{E_F-eV/2}^{E_F+eV/2} \frac{dE}{2\pi i} \text{Tr}_{\text{spin}} [\hat{\sigma} G^<(r,r;E;V)]
\]

\[ E_F = -3.8t_o \]

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Direction and Sign of Spin Accumulation on Lateral Edges

\[ \langle S_z(x, y) \rangle \]

\[ \langle S(x, y) \rangle \]

\[ \begin{bmatrix} I^s_z \end{bmatrix} = -\begin{bmatrix} I^s_x \end{bmatrix} \begin{bmatrix} I^s_x \end{bmatrix} = -\begin{bmatrix} I^s_x \end{bmatrix} \]

\[ \begin{bmatrix} I^s_x \end{bmatrix} = \begin{bmatrix} I^s_y \end{bmatrix} \]

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Disorder and In-Plane Magnetic Field Effects

\[ \langle S_z(x=x_0, y) \rangle (10^{-3} \hbar/2) \]

\[ g*\mu_B B_y^{\text{ext}}/t_0 \]

- \[ E_F = -3.8t_0 \]
- \[ t_{SO} = 0.1t_0 \]
- \[ eV = 0.4t_0 \]

Transverse Coordinate \( y \) (\( a \))

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Transverse Spin-Orbit “Force”

\[
\hat{F}_H = m^* \frac{d^2 \mathbf{r}_H}{dt^2} = -\frac{m^*}{\hbar^2} [\mathbf{\dot{r}}_H, \hat{H}, \hat{H}] = \frac{2\alpha^2 m^*}{\hbar^3} (\mathbf{\hat{p}}_H \times \mathbf{z}) \otimes \mathbf{\hat{\sigma}}^z_H - \frac{dV_{\text{conf}}(\mathbf{\hat{y}}_H)}{d\mathbf{\hat{y}}_H} \mathbf{\hat{y}}
\]

\[
|\Psi(t = 0)\rangle = |\text{packet}\rangle \otimes |\sigma\rangle
\]

\[
\langle \Psi | \hat{F}_y | \Psi \rangle
\]

\[
\langle \Psi | \mathbf{\hat{y}} | \Psi \rangle
\]

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Deflection of spin densities by the SO "Force"
Spin Decoherence and Decaying SO “Force”

\[ \dot{\rho}_s(t) = \text{Tr}_0 |\Psi(t)\rangle\langle \Psi(t)| = \sum_m \langle m|\Psi(t)\rangle\langle \Psi(t)|m \rangle = \sum_{m,\sigma,\sigma'} c_{m,\sigma}(t)|\sigma\rangle\langle \sigma'|c_{m,\sigma}^*(t) = \frac{1}{2}(1 + P \cdot \hat{\sigma}) \]

Chang et al., PRB 70, 245309 (2004).
Spin Hall Effect as a Semiconductor Source of Pure Spin Currents

\[ I \neq 0 \quad I_s \neq 0 \]

\[ I = I_\uparrow + I_\downarrow \]

\[ I_s = \frac{\hbar}{2e}(I_\uparrow - I_\downarrow) \]
\section*{Mesoscopic Spin Hall Effect}

- Pure Spin Hall current in the transverse ideal leads:

\[ I_2 = I_2^\uparrow + I_2^\downarrow = 0 \]
\[ I_2^s = \frac{\hbar}{2e} (I_2^\uparrow - I_2^\downarrow) \neq 0 \]

\[ G_{SH} = \frac{I_2^s}{V_1 - V_4} \]

Landauer-Büttiker Approach to Multiprobe Spin Current Formulae

\[ I_p = \sum_q G_{pq} (V_p - V_q) \]

Time-reversal invariance (preserved by SO $B(k)$):

\[ G_{pq} = G_{qp} \]
\[ G_{pq}^{\sigma\sigma'} = G_{qp}^{-\sigma'\sigma} \]

Spin-resolved conductance coefficients:

\[ G_{pq}^{\text{in}} = G_{pq}^{\uparrow\uparrow} + G_{pq}^{\uparrow\downarrow} - G_{pq}^{\downarrow\uparrow} - G_{pq}^{\downarrow\downarrow} \]
\[ G_{qp}^{\text{out}} = G_{qp}^{\uparrow\uparrow} + G_{qp}^{\uparrow\downarrow} - G_{qp}^{\downarrow\uparrow} - G_{qp}^{\downarrow\downarrow} \]

\[ \Rightarrow V_p = \text{const.} \rightarrow I_p^{s} \neq 0 ? \]

PRL 94, 106602 (2005)
Mesoscopic Spin Hall Conductance: Finite-Size Scaling

The key mesoscale is the spin precession length \( L_{SO} = 2\pi \hbar^2 / m^* \alpha = \pi a t_o / 2t_{SO} \) (which also plays the role of Dyakonov-Perel spin relaxation length in disordered systems) over which spin precesses from \( \uparrow \) to \( \downarrow \): \( G_{sh}^z (L_{SO}) \approx 0.2 e/4\pi \) is the maximum value of the spin Hall conductance.

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Rashba SO coupling switched on adiabatically (via simple linear function) within finite length region of the leads.
Mesoscopic Spin Hall Conductance: Sensitivity to Impurities and Localization

\[ \langle G \rangle, \ e^{\langle \ln G \rangle} \approx 0.1 \frac{2e^2}{\hbar} \]

- **Metallic:** \[ g(L) \propto L^{d-2} \]
- **Localized:** \[ g(L) \propto e^{-L/\xi} \]
- **Critical:** \[ g(L) = g_c, \ \xi = |W - W_c|^{-\nu} \]

\[ \alpha k_F \hbar / \tau = (t_{SO} \ell) / (t_o a) \approx 0.1 \]

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Quantum Interference Driven
Spin Hall Effect

\[ L < L_\phi \Rightarrow |\Psi\rangle \in \mathcal{H}_o \otimes \mathcal{H}_s \]

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All-Electrical Detectors for Quantum-Coherent Spin Hall Flow in Rings

\[ E_F = -0.05, Q_R^B = 4.3 \]

\[ N_A = N_B = 100, N_C = 5 \]

\[ (V_{5,6} - V_5) N_1 G_z^{sH} = I_z^s N_2 E_{1/2} (e/8\pi) \]

Rashba SO coupling \( Q_R^A \)
Local Spin Hall Fluxes: Equilibrium vs. Nonequilibrium

\[
\langle \hat{J}^{S_i}_{mm'} \rangle = \frac{t_o}{2} \int_{-\infty}^{\infty} \frac{dE}{2\pi} \text{Tr}_s \left[ \hat{\sigma}_i \left( G^\leq_{m'm}(E) - G^\leq_{mm'}(E) \right) \right]
\]

\[+ \left[ e_i \times (m' - m) \right]_z \frac{t_{SO}}{2} \int_{-\infty}^{\infty} \frac{dE}{2\pi i} \text{Tr}_s \left[ G^\leq_{mm'}(E) + G^\leq_{m'm}(E) \right] \]

Equilibrium

Nonequilibrium

\( eV=0 \)

\( t_{SO} = 0 \)

\( t_{SO} = 0.1 t_o \)

\( \pm eV/2 \)

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\[
\langle j_{mm'}^S \rangle = \frac{t_0}{2} \int_{E_b}^{E_F-eV/2} \frac{dE}{2\pi} \text{Tr}_s [\hat{\sigma}_z (G_{m'm}(E) - G_{mm'}^{\leq}(E))] + \frac{t_0}{2} \int_{E_F-eV/2}^{E_F+eV/2} \frac{dE}{2\pi} \text{Tr}_s [\hat{\sigma}_z (G_{m'm}^{\leq}(E) - G_{mm'}^{\leq}(E))] \\
= \langle j_{mm'}^{S_{eq}} \rangle + \langle j_{mm'}^{S_{neq}} \rangle.
\]
Nonconservation of Spin Current Density Inside the SO Coupled Medium

\[ I^{S_z}_{\text{trans}}(m_y) = \sum_{m_x} \left\langle \hat{J}^{S_z}_{m_x,m_y}(m_x,m_y+1) \right\rangle \]
Rashba’s In-Plane Polarized Equilibrium Spin Fluxes: $\mathcal{J}_x = \mathcal{J}_y \equiv 0, \mathcal{J}_x = -\mathcal{J}_y$

$$
\langle \hat{J}_{m m'}^{S_z} \rangle = \frac{t_o}{2} \int_{E_b}^{E_F} \frac{dE}{2\pi i} \text{Tr}_s \left[ \hat{\sigma}_z \left( \mathbf{G}_{m' m}(E) - \mathbf{G}_{m m'}^{<}(E) \right) \right]
$$

$$
+ \left[ \mathbf{e}_i \times (\mathbf{m}' - \mathbf{m}) \right]_z \frac{t_{SO}}{2} \int_{E_b}^{E_F} \frac{dE}{2\pi i} \text{Tr}_s \left[ \mathbf{G}_{m m'}^{<}(E) + \mathbf{G}_{m' m}^{<}(E) \right]
$$

$$
\langle \hat{J}_{m m'}^{S_x} \rangle = 0
$$

$$
\langle \hat{J}_{m m'}^{S_y} \rangle = 0
$$

$$
I_{trans}^{S_x(eq)}(m_y) = -I_{long}^{S_y(eq)}(m_x)
$$
Experimental Imaging of Charge and Spin Flow

Steady-State Local Spin Density

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Mesoscopic vs. Macroscopic Regime

$L \lesssim L_{SO}$ vs. $L \gg L_{SO}$
In-Plane Polarized Spin Flow
Edge vs. Bulk Spin Hall Current Density in Diffusive Rashba 2DEGs

\[ j_y^z = -\frac{eE_x}{2\pi} \zeta^2 e^{-x/L_s} \]

Mishchenko et al., PRL 93, 226602 (2004).

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Concluding Remarks

- Pure spin current, flowing in the transverse direction in response to longitudinal unpolarized quantum-coherent transport of charge through experimentally relevant system (Hall bar consisting of a finite-size Rashba spin-split 2DEG attached to four current and voltage probes), should display the following observable features:
  - It is non-zero within a finite interval of the Rashba SO coupling values,
  - The polarization vector of the flowing spin is not orthogonal to 2DEG,
  - It is determined on the processes (SO transverse force) happening on the mesoscale set by the spin precession length,
  - It gradually decays with increasing disorder, reaching zero before the onset of strongly localized phase.

- The intrinsic SO coupling driven spin Hall effect dichotomy—mesoscopic (in inhomogeneous finite-size structures) versus macroscopic (a semiclassical phenomenon driven by external electric field penetrating an infinite semiconductor)—originates from the sensitivity of the dynamics of transported spin in SO coupled semiconductors to finite-size, macroscopic inhomogeneities, and possible quantum-coherent effects.