Echoes of special relativity in condensed matter physics: anomalous Hall effect, spin-helix transistors, and topological thermoelectrics

Universität zu Köln
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JAIRO SINIOVA
Texas A&M University
Institute of Physics ASCR

Hitachi Cambridge
Joerg Wünderlich, A. Irvine, et al

Institute of Physics ASCR
Tomas Jungwirth, Vít Novák, et al

University of Würzburg
Laurens Molenkamp, E. Hankiewicz, et al

University of Nottingham
Bryan Gallagher, Richard Campion, et al.

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The electron:
the key character with dual personalities

**CHARGE**
Easy to manipulate:
Coulomb interaction

**SPIN 1/2**
Makes the electron antisocial: a fermion

Quantum mechanics

\[ E = \frac{p^2}{2m} \]
\[ E \rightarrow i\hbar \frac{d}{dt} \]
\[ p \rightarrow -i\hbar \frac{d}{dr} \]

Special relativity

\[ E^2/c^2 = p^2 + m^2 c^2 \]
(E=mc^2 for p=0)

Dirac equation

\[ i\hbar \frac{\partial}{\partial t} \Psi(x, t) = (c\vec{\alpha} \cdot \vec{p} + \beta mc^2) \Psi(x, t) \]

"Classical" external manipulation of charge & spin

Nanoelectronics, spintronics, and materials control by spin-orbit coupling
Internal communication between spin and charge: spin-orbit coupling interaction

(one of the few echoes of relativistic physics in the solid state)

Classical explanation (in reality it arises from a second order expansion of Dirac equation around the non-relativistic limit)

- “Impurity” potential $V(r)$ produces an electric field

\[ \vec{E} = -\frac{1}{e} \nabla V(\vec{r}) \]

- Motion of an electron

In the rest frame of an electron the electric field generates an effective magnetic field

\[ \vec{B}_{eff} = -\frac{\hbar \vec{k}}{cm} \times \vec{E} \]

This gives an effective interaction with the electron’s magnetic moment

\[ H_{SO} = -\vec{\mu}_B \cdot \vec{B}_{eff} \]
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Consequence #1: 
Spin or the band-structure Bloch states are linked to the momentum.

\[
H_{SO} = -\vec{\mu}_B \cdot \vec{B}_{eff}
\]
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\[ H_{SO} = -\vec{\mu}_B \cdot \vec{B}_{eff} \]

Consequence #2
Mott scattering

\[ \vec{B}_{eff} = -\frac{\hbar k}{cm} \times \vec{E} \]
Control of materials and transport properties via spin-orbit coupling

- Nanoelectronics
- Spintronics
- Materials control by spin-orbit coupling

- Nano-transport
- Magnetotransport
- Effects of spin-orbit coupling in multiband systems
- Caloritronics
- Spintronic Hall effects
- New magnetic materials
- Topological transport effects
Control of materials and transport properties via spin-orbit coupling

Anomalous Hall effects

majority

minority

Nagaosa, Sinova, Onoda, MacDonald, Ong, RMP 10

Anomalous Hall Effect: the basics

Spin dependent “force” deflects **like-spin** particles

\[ \rho_{xy} = -\frac{\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2} \approx -\frac{\sigma_{xy}}{\sigma_{xx}^2} \approx -\sigma_{xy} \rho_{xx}^2 \approx -A \rho_{xx} - B \rho_{xx}^2 \]

\[ \sigma^{\text{AH}}_{xy} \approx B + A \sigma_{xx} \]

AHE is does NOT originate from any internal magnetic field created by \( M_\perp \); the field would have to be of the order of 100T!!!
Anomalous Hall effect (scaling with $\rho$ for metals)

$$\rho_{xy} = -A\rho_{xx} - B\rho_{xx}^2$$

$$\sigma_{xy} \approx B + A\sigma_{xx}$$

$\sigma_{xx} > 10^6 \, (\Omega \text{cm})^{-1}$

$\sigma_{xx} \sim 10^4-10^6 \, (\Omega \text{cm})^{-1}$

Material with dominant skew scattering mechanism

Material with dominant scattering-independent mechanism

Nanoelectronics, spintronics, and materials control by spin-orbit coupling
Anomalous Hall effect (scaling for insulators)

\[ \sigma_{xy} \sim \sigma_{xx}^{1.4-1.7} \text{ over a few decades for } \sigma_{xx} < 10^4 \, (\Omega \text{cm})^{-1} \]

Diagonal hopping conductivity for most systems showing approximate scaling.
Tentative phase diagram of AHE

Nagaosa et al RMP 10
Anomalous Hall effect: more than meets the eye

Anomalous Hall Effect

Spin Hall Effect

Topological Insulators

Inverse SHE

Mesoscopic Spin Hall Effect

Spin-injection Hall Effect

Nanoelectronics, spintronics, and materials control by spin-orbit coupling
Cartoon of the mechanisms contributing to AHE \( \sigma_{xy}^{AH} \approx B + A\sigma_{xx} \)

**Skew scattering**  
\( A \sim 1/n_i \)  
Asymmetric scattering due to the spin-orbit coupling of the electron or the impurity. Known as Mott scattering.

**Intrinsic deflection**  
\( B \)  
Electrons deflect to the right or to the left as they are accelerated by an electric field ONLY because of the spin-orbit coupling in the periodic potential (electronics structure).

**Side jump scattering**  
\( B \)  
Electrons deflect first to one side due to the field created by the impurity and deflect back when they leave the impurity since the field is opposite resulting in a side step. They however come out in a different band so this gives rise to an anomalous velocity through scattering rates times side jump.

**Electrons have an “anomalous” velocity perpendicular to the electric field related to their Berry’s phase curvature which is nonzero when they have spin-orbit coupling.**

\[ \dot{x}_c = \frac{\partial \varepsilon(k)}{\hbar \partial k} + \frac{e}{\hbar} \vec{E} \times \vec{\Omega} \]

\[ \Omega_z(k, n) = 2\text{Im} \left( \langle \frac{\partial}{\partial y} n\vec{k} | \frac{\partial}{\partial x} n\vec{k} \rangle \right) \]

**SO coupled quasiparticles**

**V_{imp}(r) (\Delta so>\hbar/\tau) or \propto \lambda^* \nabla V_{imp}(r) (\Delta so<\hbar/\tau)**
The tumultuous history of AHE

• 1880-81: Hall discovers the Hall and the anomalous Hall effect

• 1954: Karplus and Luttinger attempt first microscopic theory: they develop (and later Kohn and Luttinger) a microscopic theory of linear response transport based on the equation of motion of the density matrix for non-interacting electrons, $\frac{d\hat{\rho}}{dt} = i\frac{\hbar}{2}\left[\hat{\rho}, H_0 + V_{\text{ext}} + e\vec{E} \times \vec{\rho}\right]$; run into problems interpreting results since some terms are gauge dependent. Lack of easy physical connection.

• 1955-58: Smit attempts to create a semi-classical theory using wave-packets formed from Bloch band states: identifies the skew scattering and notices a side-step of the wave-packet upon scattering and accelerating. $\vec{r}_s(\vec{k},t) = \frac{\partial E_{\vec{k}}}{\partial \hbar k} t + \left< u_{\vec{k}} \left| \frac{\partial}{\partial \vec{k}} u_{\vec{k}} \right> \right.$ Speculates, wrongly, that the side-step cancels to zero. The physical interpretation of the cancellation is based on a gauge dependent object!!

• 1970: Berger reintroduces (and renames) the side-jump: claims that it does not vanish and that it is the dominant contribution, ignores intrinsic contribution. (problem: his side-jump is gauge dependent)

• 1970’s: Berger, Smit, and others argue about the existence of side-jump: the field is left in a confused state. Who is right? How can we tell? Three contributions to AHE are floating in the literature of the AHE: anomalous velocity (intrinsic), side-jump, and skew contributions.
The tumultuous history of AHE: last three decades

• 1980’s: Ideas of geometric phases introduce by Berry; QHE discoveries

• 2000’s: Materials with strong spin-orbit coupling show agreement with the anomalous velocity contribution: intrinsic contribution linked to Berry’s curvature of Bloch states. Ignores disorder contributions.

\[ \mathbf{r}'_c = \frac{1}{\hbar} \frac{\partial E_n(\mathbf{k}_c)}{\partial \mathbf{k}_c} + e\mathbf{E} \times \mathbf{\Omega}_{\mathbf{k}_c} \]

• 2004’s: Spin-Hall effect is revived by the proposal of intrinsic SHE (from two works working on intrinsic AHE): AHE comes to the masses, many debates are inherited in the discussions of SHE.

• 2005-10’s: Linear theories treating SO coupling and disorder finally merge: full semi-classical theory developed and microscopic approaches are in agreement among each other in simple models.
Contributions understood in simple metallic 2D models

Kubo microscopic approach: in agreement with semiclassical
Borunda, Sinova, et al PRL 07, Nunner, Sinova, et al PRB 08

$\sigma_{xy} = \sigma_{xy}^{\text{II}} + \sigma_{xy}^{\text{side jump}} + \sigma_{xy}^{\text{skew scattering}}$

Semi-classical approach:  
Gauge invariant formulation  
Sinitsyn, Sinova, et al PRB 05, PRL 06, PRB 07

Non-Equilibrium Green’s Function (NEGF) microscopic approach

$\sigma_{xy}^{\text{AH}} \approx B + A \sigma_{xx}$

$G^R = G_0 + G_0 \Sigma_R G^R$
$(G_0^{-1} - \Sigma_R)G^R = 1$
$\hat{\Sigma}^< = \hat{\Sigma}^R \otimes \hat{G}^< \otimes \hat{\Sigma}^A$

Restriction to homogeneous magnetization.
Magnetic textures lead to the so-called topological AHE.
Generalization to 3D: scattering in dependent AHE

Step 1. Use linearized version of Keldysh formalism to obtain

\[
\begin{align*}
\dot{j}_i^I &= -\frac{e^2}{\hbar} \int \frac{d^n k}{(2\pi)^n} d\omega \frac{E}{2\pi} \text{Tr} \left[ \mathcal{V} \hat{G}_{eq}^{R} \hat{\rho} \hat{G}_{eq}^{A} \hat{\nu}_i + \left( \hat{G}_{eq}^{R} \hat{\nu} \hat{G}_{eq}^{A} - \hat{G}_{eq}^{A} \hat{\nu} \hat{G}_{eq}^{A} + \hat{G}_{eq}^{R} \hat{\nu} \hat{G}_{eq}^{R} \right) / 2 \right] \hat{\nu}_i \\
\dot{j}_i^{II} &= \frac{e^2}{\hbar} \int \frac{d^n k}{(2\pi)^n} d\omega \frac{E n_F}{2\pi} \text{Tr} \left[ \mathcal{V} \hat{G}_{eq}^{R} \hat{\rho}_E^{R} \hat{G}_{eq}^{R} \hat{\nu}_i + \left( \hat{G}_{eq}^{R} \hat{\nu} \partial_\omega \hat{G}_{eq}^{R} - \partial_\omega \hat{G}_{eq}^{R} \hat{\nu} \hat{G}_{eq}^{R} \right) \hat{\nu}_i / 2 \right] + \text{c.c.}
\end{align*}
\]

where

\[
\hat{\rho} = \int \frac{d^n k}{(2\pi)^n} \left( \mathcal{V} \hat{G}_{eq}^{R} \hat{\rho} \hat{G}_{eq}^{A} + \hat{G}_{eq}^{R} \hat{\nu} \hat{G}_{eq}^{A} \right)
\]

\[
\hat{\rho}_E^{R} = \int \frac{d^n k}{(2\pi)^n} \left[ \mathcal{V} \hat{G}_{eq}^{R} \hat{\rho}_E^{R} \hat{G}_{eq}^{R} + \left( \hat{G}_{eq}^{R} \hat{\nu} \partial_\omega \hat{G}_{eq}^{R} - \partial_\omega \hat{G}_{eq}^{R} \hat{\nu} \hat{G}_{eq}^{R} \right) / 2 \right]
\]

Kovalev, Sinova, Tserkovnyak PRL 2010
Scattering-independent contribution to the AHE

\[ \sigma_{ij}^{\text{int}} = \frac{e^2}{\hbar} \sum_{\eta} \frac{d^n k}{(2\pi)^n} \frac{d\omega}{2\pi} n_F i [\hat{A}_{k_i} \hat{A}_{k_j} - \hat{A}_{k_j} \hat{A}_{k_i}]_{\eta \eta} \hat{A}_{\eta} \]

expressed through band structure only!!

Well known intrinsic contribution
Side jump contribution related to Berry curvature

\[ \hat{G}^{R(A)}_{\text{eq}} = (\omega \hat{1} - \hat{H}_0 - \hat{\Sigma}^{R(A)}_{\text{eq}})^{-1}, \hat{\Sigma}^{R(A)}_{\text{eq}}(\omega) = \mp i V_2 \hat{\gamma}^{R(A)} \]

\[ \hat{\gamma}^{R(A)}(\omega) = \pm i \int \frac{d^n k}{(2\pi)^n} \hat{G}^{R(A)}_{\text{eq}}(k, \omega) \]

\[ \hat{\gamma}_c(k, \omega) = \hat{\gamma}^{d}(k, \omega) \]

\[ \hat{G}^R_0 = (\omega \hat{1} - \hat{\epsilon}(k) \pm i V_2 \hat{\gamma}^{d}(k, \omega))^{-1} \]

\[ \hat{G}^R_c = \hat{U}^\dagger \hat{G}^{R(A)}_{\text{eq}} \hat{U} = \left[ 1 - \hat{G}^R_0 (\mp i V_2 \hat{\gamma}^{d}_c) \right]^{-1} \hat{G}^{R(A)}_0 \]

\[ \hat{P} = \int \frac{d^n k}{(2\pi)^n} \frac{1}{2 | \partial_{k} \epsilon_{\eta} | [\hat{\gamma}_{c}]_{\eta \eta}} \]

Remaining side jump contribution
Kovalev, Sinova, Tserkovnyak PRL 2010

Nanoelectronics, spintronics, and materials control by spin-orbit coupling
Results for Luttinger model: simplest model of GaMnAs

\[ \hat{H}_0 = \frac{\hbar^2}{2m_e} \left[ \left( \gamma_1 + \frac{5}{2} \gamma_2 \right) k^2 - 2 \gamma_2 (\mathbf{k} \cdot \hat{\mathbf{j}})^2 \right] - \Omega \mathbf{m} \cdot \hat{\mathbf{s}} \]

Kovalev, Sinova, Tserkovnyak PRL 2010

It gives a formalism that allows for a systematic study of AHE through ab-initio calculations (ongoing collaboration with S. Blügel and Y. Mokrousov)
Tentative phase diagram of AHE

Nagaosa et al RMP 10

\[ \sigma_{xy}^{AH} \propto \sigma_{xx}^\gamma, \quad 1.4 < \gamma < 1.7, \]

Previous Theories:
S. H. Chun et al., PRL 2000; Lyanda-Geller et al PRB 2001 (Theory for Manganites; no scaling)
A. A. Burkov and L. Balents, Phys. Rev. Lett. 91, 057202 (2003), \[ \gamma = 1.0 \]
Phonon-assisted hopping between localized states

The Hamiltonian describing localized states and e-ph interaction (Holstein 1961)

\[ H = H_{part} + H_{part-ph} + H_{ph} \]

with

\[ H_h = \sum_{i\alpha} \epsilon_i c_{i\alpha}^{\dagger} c_{i\alpha} - \sum_{i\alpha j\beta} t_{i\alpha j\beta} c_{i\alpha}^{\dagger} c_{j\beta} - \sum_{i\alpha \beta} \hbar \cdot \tau_{\alpha \beta} c_{i\alpha}^{\dagger} c_{i\beta} \]

Localization:

\[ |\frac{t_{ij}}{\epsilon_i - \epsilon_j}| \ll 1 \]

Electric current between two sites:

\[ I_{ij} = G_{ij} V_{ij} + \sum_k F_{ijk} (V_{ik} + V_{jk}) \]

- \( G_{ij} \): direct conductance due to two-site hopping. Responsible for longitudinal conductance.
- \( F_{ijk} \): off-diagonal conductance due to three-site hopping.
Transverse charge transport: Three-site hopping (Holstein, 1961)

$$P_{ij}^{(m)(H)} = \frac{2\pi}{\hbar} \sum_{\text{phonon modes}} \text{Im}[A_{ij}^{(0)dir} A_{ij}^{(0)ind}] \sin \phi \delta(\epsilon_i - \epsilon_j \pm \Delta\delta_{\text{phonon}})$$

$m$: the number of real phonons included in the whole transition.

Typical two real phonon process:

Typical one real (two virtual) phonon process:

Direct hopping

Indirect hopping

Nanoelectronics, spintronics, and materials control by spin-orbit coupling
Macroscopic anomalous Hall conductivity/resistivity via percolation theory

Critical path/cluster appears when:

$$\bar{n} = \langle n(\epsilon_i) \rangle_c = 2.6 \sim 2.7$$

$$V_y(x) = V_1^H + V_2^H + \ldots + V_N^H(x)$$

In the thermodynamic limit, we get the AHC:

$$\sigma_{xy}^{AH} = \frac{6L\sigma_{xx}^2}{N_F} \int d\epsilon_1 d\epsilon_2 d\epsilon_3 \int d^3 \vec{r}_{12} \int d^3 \vec{r}_{23} \rho(\epsilon_1)[n(\epsilon_1)]^3 \rho(\epsilon_2)[n(\epsilon_2)]^3 \rho(\epsilon_3)[n(\epsilon_3)]^3$$

$$\times \frac{G_{123}}{G_{12}G_{23} + G_{13}G_{23} + G_{31}G_{12}}$$

Unfortunately, too complicated to get an analytical calculation!!!
The lower and upper limits are given by (note the differences in the constraints):

\[
\begin{align*}
\{\sigma_{xy}^{AH}\}_{\text{min}} &= 3L\sigma_{xx}^2 < \epsilon^{2\alpha(R_{ij}+R_{jk}-R_{ik})} >_c \frac{1}{2kT}(\epsilon_i+\epsilon_j+\epsilon_k) \\
\{\sigma_{xy}^{AH}\}_{\text{max}} &= 3L\sigma_{xx}^2 < \epsilon^{2\alpha(R_{ij}+R_{jk}-R_{ik})} >_c \frac{1}{2kT}(\epsilon_i+\epsilon_j+\epsilon_k) \\
\end{align*}
\]

For the Mott hopping, we find

\[
\begin{align*}
\{\sigma_{xy}^{AH}\}_{\text{min}} &= 3L\sigma_0^{0.242} \frac{\text{Im}[\text{tr}(tkt^*j^*)]}{\beta e^2(t_{\text{max}})} \sigma_{xx}^{1.758} \propto \sigma_{xx}^{\gamma}, \quad \gamma \simeq 1.76 \\
\{\sigma_{xy}^{AH}\}_{\text{max}} &= 3L\sigma_0^{0.621} \frac{\text{Im}[\text{tr}(tkt^*j^*)]}{\beta e^2(t_{\text{min}})} \sigma_{xx}^{1.379} \propto \sigma_{xx}^{\gamma}, \quad \gamma \simeq 1.38
\end{align*}
\]

\[
\sigma_{xy}^{AH} \propto \sigma_{xx}^{\gamma}, \quad 1.38 < \gamma < 1.76.
\]

Physically, the upper limit ($\gamma=1.38$) corresponds to the situation that most triads in the system are equilateral triangles (this is actually a regular distribution), while for the lower limit ($\gamma=1.76$) the geometry of triads is randomly distributed. Since the impurity sites are randomly distributed, we expect the realistic result is usually close to the lower limit in most systems.

Xiong-Jun Liu, Sinova, unpub. 2010
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Towards a realistic spin-based non-magnetic FET device

Can we achieve direct spin polarization injection, detection, and manipulation by electrical means in an all paramagnetic semiconductor system?

**Long standing paradigm: Datta-Das FET (1990)**
Exploiting the large Rashba spin-orbit coupling in InAs

Electrons are confined in the z-direction in the first quantum state of the asymmetric trap and free to move in the x-y plane.

\[ \vec{B}_{eff} = -\frac{\hbar k}{cm} \times \vec{E} \]

Rashba effective magnetic field
Towards a realistic spin-based non-magnetic FET device

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Long standing paradigm: Datta-Das FET (1990)
Exploiting the large Rashba spin-orbit coupling in InAs

\[ \vec{B}_{eff} = -\frac{\hbar \vec{k}}{cm} \times \vec{E} \]

**BUT** \( I_{MF} \ll L_{S-D} \) at room temperature
Dephasing of the spin through the Dyakonov-Perel mechanism

\[ \vec{B}_{\text{eff}} = -\frac{\hbar \vec{k}}{cm} \times \vec{E} \]

\( L_{SD} \sim \mu m \)

\( l_{MF} \sim 10 \text{ nm} \)
Problem: Rashba SO coupling in the Datta-Das SFET is used for manipulation of spin (precession) BUT it dephases the spin too quickly.

New paradigm using SO coupling: SO not so bad for dephasing

\[ \vec{B}_{eff} = -\frac{\hbar k}{cm} \times \vec{E} \]

1) Can we use SO coupling to manipulate spin AND increase spin-coherence?

2) Can we detect the spin in a non-destructive way electrically?
Spin-dynamics in 2D electron gas with Rashba and Dresselhauss spin-orbit coupling

1) Can we use SO coupling to manipulate spin AND increase spin-coherence?

A 2DEG is well described by the effective Hamiltonian:

\[
\hat{H}_{2\text{DEG}} = \frac{\hbar^2 k_y^2}{2m} + \alpha (k_y \sigma_x - k_x \sigma_y) + \beta (k_x \sigma_x - k_y \sigma_y) + \lambda^* \vec{\sigma} \times (\vec{k} \times \nabla V_{\text{dis}}(\vec{r}))
\]

\[
\lambda^* = \frac{P^2}{3} \left( \frac{1}{E_g^2} - \frac{1}{(E_g + \Delta_{so})^2} \right) \approx 5.3 \text{Å}^2 \text{ for GaAs}
\]

\[
\beta = -B \langle k_z^2 \rangle, \quad B = 10 \text{ eV Å}^3, \quad \alpha = \lambda^* E_z
\]

- **Rashba**: from the asymmetry of the confinement in the z-direction
  - \( \alpha > 0, \beta = 0 \)

- **Dresselhauss**: from the broken inversion symmetry of the material, a bulk property
  - \( \alpha = 0, \beta < 0 \)

Nanoelectronics, spintronics, and materials control by spin-orbit coupling
Spin-dynamics in 2D electron gas with Rashba and Dresselhauss spin-orbit coupling

Something interesting occurs when $\alpha \sim -\beta$

$$H_{2DEG} \approx \frac{\hbar^2 k^2}{2m} + \alpha (k_y - k_x)(\sigma_x + \sigma_y)$$

- spin along the [110] direction is conserved
- long lived precessing spin wave for spin perpendicular to [110]

The nesting property of the Fermi surface:

$$E_\downarrow(\vec{k}) = E_\uparrow(\vec{k} + \vec{Q})$$

$$Q = \frac{4m\alpha}{\hbar^2}$$

Bernevig et al PRL 06, Weber et al. PRL 07

Schliemann et al PRL 04
Effects of Rashba and Dresselhaus SO coupling

\[ H_{2\text{DEG}} = \frac{\hbar^2 k^2}{2m} + \alpha (k_y \sigma_x - k_x \sigma_y) + \beta (k_x \sigma_x - k_y \sigma_y) \]

\( \alpha > 0, \beta = 0 \)

\( \alpha = 0, \beta < 0 \)

\( \alpha = -\beta \)
Spin-dynamics in 2D systems with Rashba and Dresselhauss SO coupling

For the same distance traveled along [1-10], the spin precesses by exactly the same angle.
Persistent state spin helix verified by pump-probe experiments

Nondiffusive Spin Dynamics in a Two-Dimensional Electron Gas


PRL 98, 076604 (2007)  PHYSICAL REVIEW LETTERS  week ending 16 FEBRUARY 2007

Similar wafer parameters to ours

Nanoelectronics, spintronics, and materials control by spin-orbit coupling
Spin-helix state when $\alpha \neq \beta$

\[
S_z/x - (x[1\bar{1}0]) = S^0_z/x - \exp[qx[1\bar{1}0]]
\]

\[
|q| = \left(\tilde{L}_1^2\tilde{L}_2^2 + \tilde{L}_4^4\right)^{1/4} \text{ and } \theta = \frac{1}{2} \arctan\left(\frac{\sqrt{2\tilde{L}_1^2\tilde{L}_2^2 - \tilde{L}_4^4/4}}{\tilde{L}_2^2 - \tilde{L}_1^2/2}\right)
\]

\[
\tilde{L}_{1/2} = 2m|\alpha \pm \beta|/\hbar^2
\]

For Rashba or Dresselhaus by themselves NO oscillations are present; only and over damped solution exists; i.e. the spin-orbit coupling destroys the phase coherence.

There must be TWO competing spin-orbit interactions for the spin to survive!!!
New paradigm using SO coupling: SO not so bad for dephasing

Problem: Rashba SO coupling in the Datta-Das SFET is used for manipulation of spin (precession) BUT it dephases the spin too quickly (DP mechanism).

1) Can we use SO coupling to manipulate spin AND increase spin-coherence? ✓

Use the persistent spin-Helix state and control of SO coupling strength (Bernevig et al 06, Weber et al 07, Wünderlich et al 09)

2) Can we detect the spin in a non-destructive way electrically
Two types of contributions:

i) S.O. from band structure interacting with the field (external and internal)

ii) Bloch electrons interacting with S.O. part of the disorder

Type (i) contribution much smaller in the weak SO coupled regime where the SO-coupled bands are not resolved, dominant contribution from type (ii)

\[
H_{2DEG} = \frac{\hbar^2 k^2}{2m} + \alpha(k_y \sigma_x - k_x \sigma_y) + \beta(k_x \sigma_x - k_y \sigma_y) + \lambda^* \vec{\sigma} \times (\vec{k} \times \nabla V_{dis}(\vec{r}))
\]

\[
\left| \sigma_{xy} \right|_{\text{skew}} = \frac{2\pi e^2 \lambda^*}{\hbar^2} V_0 \tau n(n_\uparrow - n_\downarrow)
\]

\[
\left| \sigma_{xy} \right|_{\text{side-jump}} = \frac{2e^2 \lambda^*}{\hbar} (n_\uparrow - n_\downarrow)
\]

\[
\alpha_H(x_{[1\bar{1}0]}) = 2\pi \lambda^* \sqrt{\frac{e}{\hbar n_i \mu}} n p_z(x_{[1\bar{1}0]}) \approx 1.1 \times 10^{-3} p_z
\]

Wunderlich, Irvine, Sinova, Jungwirth, et al, Nature Physics 09

Crepieux et al PRB 01
Nozier et al J. Phys. 79

Lower bound estimate of skew scatt. contribution
Spin-injection Hall effect: theoretical expectations

Local spin-polarization → calculation of AHE signal
Weak SO coupling regime → extrinsic skew-scattering term is dominant

\[ \alpha_H(x_{[1\bar{1}0]}) = 2\pi \lambda^* \frac{e}{\hbar n_i \mu} n_{p_z}(x_{[1\bar{1}0]}) \]

Lower bound estimate

1) Can we use SO coupling to manipulate spin AND increase spin-coherence?
   Use the persistent spin-Helix state and control of SO coupling strength ✓

2) Can we detect the spin in a non-destructive way electrically?
   Use AHE to measure injected current polarization electrically ✓

Nanoelectronics, spintronics, and materials control by spin-orbit coupling
Spin-injection Hall device measurements

Wunderlich, Irvine, Sinova, Jungwirth, et al, Nature Physics 09

Nanoelectronics, spintronics, and materials control by spin-orbit coupling
Further experimental tests of the observed SIHE
SiHE: new results

Spin Hall effect transistor:
SiHE transistor

SHE transistor AND gate

Nanoelectronics, spintronics, and materials control by spin-orbit coupling
Echoes of special relativity in condensed matter physics: anomalous Hall effect, spin-helix transistors, and topological thermoelectrics

I. Introduction: using the dual personality of the electron
   • Electronics, ferromagnetism, and spintronics
   • Internal coupling of charge and spin: origin and present use
   • Control of material and transport properties through spin-orbit coupling
   • Overview of program

II. Anomalous Hall effect: from the metallic to the insulating regime
   • Anomalous Hall effect basics, history, progress in the metallic regime
   • Anomalous Hall effect in the hopping regime

III. Spin injection Hall effect: a new paradigm in exploiting SO coupling
   • Spin based FET: old and new paradigm in charge-spin transport
   • Theory expectations and modeling
   • Experimental results

IV. Topological thermoelectrics:
   • Thermoelectric figure of merit
   • Increase of ZT in topological insulators.
Control of materials and transport properties via spin-orbit coupling

- **Nano-transport**
- **Magneto-transport**
- **New magnetic materials**
- **Effects of spin-orbit coupling in multiband systems**
- **Caloritronics**
- **Topological thermoelectrics**
- **Spintronic Hall effects**

Nanoelectronics, spintronics, and materials control by spin-orbit coupling
From AHE to topological insulators to thermoelectrics

Topological Insulators: edge (2D) or surface states (3D) survive disorder effects when the bulk gap is produced by spin-orbit coupling.

Kane, Zhang, Molenkamp, Moore, et al

Dislocations have 1D channels which also protected

Vishwanath et al Nature Physics 09

Zhang, Physics 1, 6 (2008)
Nanoelectronics, spintronics, and materials control by spin-orbit coupling

From AHE to topological insulators to thermoelectrics

Seebeck coefficient

\[ S = \frac{\pi k^2 T}{3e} \left[ \frac{\partial \ln \sigma(E)}{\partial E} \right]_{E_F} \]

\[ ZT = \frac{\sigma S^2}{\kappa_e + \kappa_l} T \]

Dislocations have 1D channels which also protected

\[ \kappa_l \downarrow \sigma \uparrow S \uparrow \]

Can we obtain high ZT through the topological protected states; are they related to the high ZT of these materials?

Vishwanath et al 09
Possible large ZT through dislocation engineering

Large thermoelectric figure of merit for three-dimensional topological Anderson insulators via line dislocation engineering

O. A. Tretiakov, Ar. Abanov, Shuichi Murakami, and Jairo Sinova
(Received 23 July 2010; accepted 30 July 2010; published online 18 August 2010)

\[
\frac{1}{ZT} = \frac{(L^b_0 + snL^{1D}_0)(L^b_2 + snL^{1D}_2 + \kappa_{ph}T)}{(L^b_1 + snL^{1D}_1)^2} - 1
\]

where the L's are the linear Onsager dynamic coefficients

\[
L^{1D}_\alpha = -\frac{l}{sh} \int \mathcal{T}(E) f'(E)(E - \mu)^\alpha dE
\]

\[
L^b_\alpha = -\tau \int^{\infty}_{E_m} D(E)f'(E)v^2(E - \mu)^\alpha dE
\]

\[
L^b_\alpha = \frac{2\sqrt{2m^*}}{\pi^2 \hbar^3} \tau cT^{\alpha+3/2} \int^{\infty}_{E_m=\mu} dx \frac{x^\alpha(x + \mu/T)^{3/2}e^x}{(e^x + 1)^2}
\]

\[\text{Bi}_{1-x}\text{Sb}_x (0.07 < x < 0.22)\]

Localized bulk states

Tretiakov, Abanov, Murakami, Sinova APL 2010

Nanoelectronics, spintronics, and materials control by spin-orbit coupling
Possible large ZT through dislocation engineering

Remains very speculative but simple theory gives large ZT for reasonable parameters

Tretiakov, Abanov, Murakami, Sinova APL 2010
I. Anomalous Hall effect: from the metallic to the insulating regime
   • Established a consistent theory of Anomalous Hall effect for metallic regime with homogeneous magnetization
   • Theory of observed scaling in Anomalous Hall effect in the hopping regime
II. Spin injection Hall effect: a new paradigm in exploiting SO coupling
   • Modeled and constructed a spin FET in the diffusive regime
   • First spin AND gate with pure spin currents
III. Topological thermoelectrics:
   • Speculative theory of how to increase of ZT in topological insulators via line dislocations.

A long list of challenges: DMSs, Antiferromagnetic semiconductors, current driven magnetization dynamics, pseudo-spintronics in double layer systems, spin-caloritronics (Spin Seebeck effect)
Nanoelectronics, spintronics, and materials control by spin-orbit coupling

Sinova’s group

Principal Collaborators

Ewelina Hankiewicz
(Texas A&M Univ.)
Würzburg University

Tomas Jungwirth
Texas A&M U.
Inst. of Phys. ASCR
U. of Nottingham

Allan MacDonald
U of Texas

Joerg Wunderlich
Cambridge-Hitachi

Laurens Molenkamp
Würzburg

Bryan Gallagher
U. of Nottingham

Gerrit Bauer
TU Delft

and many others

Mario Borunda
Texas A&M Univ.
Harvard Univ.

Alexey Kovalev
Texas A&M U.
UCLA

Nikolai Sinitsyn
Texas A&M U.
U. of Texas
LANL

Xiong-Jun Liu
Texas A&M U.

Oleg Tretiakov
Texas A&M U.

Liviu Zarbo
Texas A&M Univ.

Xin Liu
Texas A&M U.

Xiong-Jun Liu
Texas A&M U.