Quadrupole Gravity Wave Estimate: Comparison with LIGO Results

Wayne M. Saslow

Department of Physics, Texas A&M University, College Station, TX 77840-4242

We apply the equation for weak gravitational radiation due to a quadrupole moment to the recent astonishing measurements of LIGO, where the collapse of a black hole binary pair into a single, larger black hole, leads to an energy loss relative to rest mass of about 0.05. This is easily obtained using values for the characteristic frequency \( f_c \) and quadrupolar distance \( R_Q \) taken from the LIGO paper itself.

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I. INTRODUCTION

Recently, gravitational radiation appears to have been directly observed simultaneously in two different laboratories.\(^1\) It is a colossal vindication of Einstein’s theory of gravity, also known as the General Theory of Relativity.\(^2\) It is also an even more monumental technical achievement.

The above work gives a range of characteristic frequencies \( f_c \), black hole mass \( M \), and quadrupolar distance \( R_Q \) associated with the self-orbiting motions of the two black holes of nearly the same mass. The details of the operation of the detectors are certainly beyond the current expertise of the present author. However, on the basis of the published ranges for \( f_c \), \( M \), and \( R_Q \),\(^1\) we note that the simple formula for quadrupolar radiation, given in Ref. 3, gives a value for the radiated energy divided by the initial rest mass energy that is quite reasonable.

II. RADIATION ESTIMATE

Specifically, eq.(105.12) of Ref. 3 gives the formula (with their \( k \) replaced by the conventional \( G = 6.67 \times 10^{-11} \) m\(^3\)/kg-s\(^2\))

\[
\frac{d\mathcal{E}}{dt}_{\text{grav}} = -\frac{G(D_{\alpha\beta})^2}{45c^5}.
\]

Here \( D_{\alpha\beta} \) has units of a mass quadrupole. Neglecting the tensor indices, we take this to be of the order of \( MR_Q^2 \), where \( M \) is one of the masses of the binary pair, taken to be \( \sim 30 \) sun masses \( M_{\odot} = 2 \times 10^{30} \) kg, and \( R_Q \) is taken to be a typical distance associated with the quadrupole moment, given as approximately \( 350 \times 10^3 \) km.

For simplicity we will assume that \( D_{\alpha\beta} \sim \omega_c^3 MR_Q^2 \). For comparison, we take the initial energy \( E_0 \) to be given by

\[
E_0 = 2Mc^2,
\]

so that

\[
\frac{1}{E_0} \left| \frac{d\mathcal{E}}{dt}_{\text{grav}} \right| = \frac{GM\omega_c^6 R_Q^4}{90c^7}.
\]

The total energy radiated is on the order of \( \omega_c^{-1} \) times this, so

\[
\frac{1}{E_0} \left| \Delta\mathcal{E} \right|_{\text{grav}} = \frac{GM\omega_c^5 R_Q^4}{90c^7}.
\]

Using the above values for \( G, M, R_Q \), and \( \omega_c = 2000 \text{s}^{-1} \), eq.(4) yields 0.049, which is pretty much what is observed.\(^1\)

III. SUMMARY

To summarize, the simple estimate of the quadrupolar gravitational radiation rate calculated above is in surprisingly close agreement with the observations. However, this result should only be taken as an indication that General Relativity is capable of explaining the observations. This result does not have any of real details that the full theory provides, and is by no means a substitute for it.

The reader is referred to Ref. 1 for an explanation of how the values we employed came about. In particular, the upper part of Fig. 2 is, in our opinion, quite extraordinary for the accuracy of the theory – to make no mention of the sensitivity of the experimental apparatus.

\(^{\ast}\) wsaslow@tamu.edu