4. (35 pts) A point charge \( Q_1 = -4.0 \times 10^{-9} \) C is on the negative \( y \)-axis at \( r_1 = 3 \) cm from the origin. A point charge \( Q_2 = 6.0 \times 10^{-9} \) C makes a counterclockwise angle \( \theta = 128^\circ \) to the positive \( x \)-axis, at \( r_2 = 4 \) cm from the origin. A charge \( Q = -3.0 \times 10^{-9} \) C is placed at the origin. \( Q_1 \) and \( Q_2 \) act on \( Q \) with forces \( F_1 \) and \( F_2 \).

a. Find \( |F_1| \) and \( |F_2| \):
\[
|F_1| = k \frac{|Q_1 Q|}{r_1^2} = \frac{(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(4 \times 10^{-9} \text{ C})(3 \times 10^{-9} \text{ C})}{(3 \text{ cm})^2} = 1.2 \times 10^{-5} \text{ N}
\]
\[
|F_2| = k \frac{|Q_2 Q|}{r_2^2} = \frac{(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(6 \times 10^{-9} \text{ C})(3 \times 10^{-9} \text{ C})}{(4 \text{ cm})^2} = 10.125 \times 10^{-5} \text{ N}
\]

b. On the figure, draw \( F_1 \) and \( F_2 \) with their tails on \( Q \), and in relative proportion.

c. Find \( F_x \), the \( x \)-component of the total force \( \vec{F} \) on \( Q \):
\[
F_x = |F_1| + |F_2| \sin 128^\circ = 5.78 \times 10^{-5} \text{ N}
\]

d. Find \( F_y \), the \( y \)-component of \( \vec{F} \):
\[
F_y = |F_2| \sin 128^\circ = 7.98 \times 10^{-5} \text{ N}
\]

e. Find the angle \( \vec{F} \) makes with respect to the \( x \)-axis, and on the figure sketch the direction of \( \vec{F} \):
\[
\tan \theta = \frac{F_y}{F_x} = 1.382, \quad \theta = 54.1^\circ \quad \text{in first quadrant, as expected}
\]

f. Find \( |\vec{F}| \):
\[
|\vec{F}| = \sqrt{F_x^2 + F_y^2} = 9.85 \times 10^{-5} \text{ N}
\]

g. \( Q_1 \) and \( Q_2 \) are rotated clockwise by 35 degrees about the origin. From parts e) and f) find the new \( F_x \):
\[
F_x' = |\vec{F}| \cos (54.1^\circ - 35^\circ) = 9.31 \times 10^{-5} \text{ N}
\]