10. (15 pts) Two identical bar magnets of length 20 cm and square pole face of area 9 cm$^2$ have magnetization $M = 0.5 \times 10^6$ A/m. Determine their pole strengths. If the magnets are placed in a line, with the N pole of one 1 cm from the S pole of the other, estimate their force of attraction. Estimate the field that the large N pole makes at the S pole of the smaller. Briefly explain your reasoning.

\[ M = \mathcal{G}_m = \frac{q_m}{A}, \quad q_m = MA = (0.5 \times 10^6 \text{ A/m})(9 \times 10^{-4} \text{ m}^2) = 450 \text{ A-m} \]

\[ |F| = \frac{k q_m^2}{r^2} = \frac{(10^{-7} \text{ N/A}^2)(450 \text{ A-m})^2}{(0.01 \text{ m})^2} = 202.5 \text{ N} \]

\[ |B| = \frac{k q_m}{r^2} = \frac{(10^{-7} \text{ N/A}^2)(450 \text{ A-m})}{(0.01 \text{ m})^2} = 0.45 \frac{\text{ N}}{\text{ A-m}} = 0.45 \text{ T} \]

We are treating the poles as points.

11. (15 pts) A 20 cm long permanent magnet has a 0.4 cm-by-0.4 cm cross-section. Its north pole is placed against a refrigerator door (made of 'soft' iron). A force of 0.05 N is required to pull the magnet off the door. Estimate its magnetization (magnetic moment per unit volume). Briefly explain your reasoning.

\[ \text{Magnet} \quad \text{iron} \quad \text{force} \]

\[ \text{1. Treat the pole faces as planes, with the response at the refrigerator being an opposite sheet of charge. Neglect the other poles.} \]

\[ |F| = |Q_A| |B_N| \]

Here \( |Q_A| = MA \) and \( |B_N| = 2 \pi k M \).

Thus \( |F| = |Q_A| |B_N| = 2 \pi k M \).

So \( M = \frac{|F|}{2 \pi k M} = \frac{0.05 \text{ N}}{2 \pi \cdot 0.01 \text{ m}} = 7.95 \times 10^4 \text{ A/m}^2 \).

12. (10 pts) A magnet of length 6 cm, mass 45 g, and magnetic moment 1.2 A-m$^2$ is in a horizontal magnetic field. Find its pole strength. If the torque on it is 0.004 N-m when its axis is at 40$^\circ$ to the field, determine the field strength.

\[ \mu = q_m \frac{M}{A}, \quad q_m = \frac{M}{A} = 20 \text{ A-m} \]

\[ |B| = \frac{|F| |B| \sin \theta}{180^\circ} = \frac{0.004 \text{ N-m}}{120 \text{ A-m}^2 \sin 40^\circ} = 5.19 \times 10^{-3} \text{ T} \]