
[2] For plane wave reflection from a conducting surface, assume that the polarization and surface orientation are as given in equations (9.140) to (9.142) in the text.

(a) Assuming that the conductivity is large enough so that the factors of 1 can be ignored in equation (9.126), show that $k$ and $\kappa$ are equal.

(b) In the limit of large conductivity, evaluate the current density in the conductor, as a function of $z$ and $t$, in terms of $\vec{E}_i$, as well as the frequency and conductivity. Assume that the conductor is surrounded by vacuum, so that $v_1 = c$.

(c) Starting along a segment of length $l$ in the $x$ direction on the surface, the current density can be integrated from $z = 0$ to $\infty$ to give the total current flowing vs. time through the region below the surface. Or, since the current is effectively confined to a narrow skin depth region, the integrated current per length can be considered a surface current density, $\vec{K}$. By this method, evaluate the surface current density vs. time.

(d) Show that the surface current you obtained satisfies the boundary condition for magnetic fields (see eqns. 9.139), considering that the interior $B$ field is zero, outside of the skin depth region.


[5] Consider the case of normal-incidence reflection of waves from a perfect conductor, with amplitude $E_o$ and frequency $\omega$.

(a) Evaluate the average radiation pressure acting on the surface.

(b) Evaluate the current density vs. time on the surface, and the magnetic field vs. time immediately adjacent to the conductor.

(c) Show that the magnetic Lorentz force per area acting on the conductor derived from these quantities has the same magnitude and direction as the radiation pressure you derived. (Note that the force on a surface current located along a discontinuity in the field is given by the average of the forces that would be exerted by the fields on each side, similar to the result for surface charges as given in equation 2.50 in the text.)

[6] In class we saw a simulation of a parallel-polarized plane wave, propagating at 45° relative to the horizontal, superposed on a reflected wave with wave-vector rotated by 90°. In one of the cases shown, the relative amplitudes were $E_R/E_I = -0.3$. (Negative means that $E_R$ points in the opposite direction from what is shown in figure 9.15.) Assume that these waves are in vacuum ($n_1 = 1$), and that the reflection occurs at the surface of a nonmagnetic dielectric.

(a) Find $n_2$, the index of the dielectric. You may solve this numerically.

(b) Determine the angle of propagation of the transmitted wave, and the relative magnitude of its $E$ field.

(c) Find the relative magnitudes of the intensities of the transmitted and reflected wave vs. the incoming wave. (Note that these will not necessarily add to 1.)