(1) Infinite parallel conducting planes are grounded, and situated at $y = 0$ and at $y = a$. The planes extend from $x = 0$ to infinity, similar to example 3.3 in the text. Solve the Laplace equation in the region of positive $x$ between planes, for the situation where the plane at $x = 0$ has the potential $V = 0$ for $0 < y < a/4$ and $3a/4 < y < a$, and $V = V_o$ for $a/4 < y < 3a/4$.

(2) Consider an $a \times a \times 2a$ rectangular box, with the $a \times a$ square faces parallel to the $y-z$ plane and located at $x = a$ and $x = -a$. Place the origin at the center of the box. The two $a \times a$ faces have constant potential $+V_o$ and $-V_o$, for at $x = a$ and $-a$ respectively, and the other 4 faces are conducting and grounded ($V = 0$), and aligned with the $x-y$ and $x-z$ planes.

[a] Solve for the potential inside the box.

[b] Find an expression for the charge density vs. position on one of the $a \times 2a$ conducting faces.

(3) A thin spherical shell (radius $R$) carries a surface charge $\sigma$ configured so that the potential at $r = R$ is given by, $V = A(\cos^2 \theta - \cos \theta + 2)$, with $A$ a constant. There is no other charge inside or outside.

[a] Solve for the potential vs. position both outside and inside the sphere.

[b] Find the charge density, $\sigma$, vs. orientation on the sphere.

(4) Griffiths problem 3.20 (page 145)

(5) [a] As stated in class, the general form of the solution to the Laplace equation in cylindrical coordinates has the form, $V = \sum_n (A_n r^n + B_n r^{-n})(C_n \cos n\phi + D_n \sin n\phi)$. Show that this function satisfies the Laplace equation, for integers $n \geq 1$.

[b] In spherical coordinates, the general solution $V = \sum_l (A_l r^l + B_l r^{-l+1})P_l(\cos \theta)$ also applies for the relatively simple case of a sphere with constant potential on its surface. Demonstrate by determining the potential vs. position on the inside of a hollow conducting spherical shell held at potential $V_o$, expressing your answer in terms of the appropriate Legendre polynomial(s).

[c] In contrast to the spherical (and Cartesian) situations, for cylindrical coordinates constant boundary conditions requires a different function than appears in the $\phi$-dependent solutions. Solve for such a $\phi$-independent function as a function of the radius; the Laplace equation in this case is an ordinary $2^{nd}$-order differential equation, so your solution should involve two adjustable parameters. [Hence, the most general solution for cases of cylindrical boundary conditions will include this general solution added to the function specified in part (a).]

[d] As an example, solve for the case of a gap between two long concentric cylindrical conducting shells. The inner shell, radius $a$, is held at potential $V_o$, while the outer shell, radius $b$, is held at potential $V_b$. Starting with your solution for part (c), solve for the potential vs. position between the two shells. Also solve for the surface charge densities on the two surfaces facing the gap region.

(6) A very long thin insulating cylindrical shell, radius $R$, carries a charge density $\sigma = \sigma_o \cos(4\phi)$. Find the electric potential inside and outside the shell, if there are no other charges in the vicinity.