(1) [7 points] A magnetic field is directed perpendicular to the plane of a 0.15-m x 0.30-m rectangular coil comprised of 320 loops of wire. To induce an average emf of \(-1.2 \text{ V}\) in the coil, the magnetic field is increased from 0.1 T to 1.5 T during a time interval \(\Delta t\). Determine \(\Delta t\).

\[
\text{flux} = B \cdot \text{Area} \cdot N, \quad \text{and} \quad \mathcal{E} = \frac{-d\Phi}{dt} = -\frac{\Delta \Phi}{\Delta t} \quad \text{(average)}
\]

so

\[-(1.2 \text{ V}) = -\frac{\Delta (B \cdot A \cdot N)}{\Delta t} = -(\Delta B) \cdot A \cdot N
\]

and

\[\Delta t = \frac{(1.4 \text{ T})(15 \text{ m} \times 0.30 \text{ m})(320)}{1.2 \text{ V}} = 16.8 \text{ s}
\]

\[\approx 17.5
\]

(2) [6 points] Frank's Aggie ring which is 2.3 cm high is placed 20 cm away from a converging lens having a focal length of 35 cm. Find the height of the image. Is this a real or virtual image?

\[\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} \rightarrow d_i = \frac{f \cdot d_o}{d_o - f}
\]

\[\begin{align*}
\text{negative means on object side} \rightarrow \text{virtual image} \\
h_i = h_o \frac{d_i}{d_o} = 3.4 \text{ cm (upright image)}
\end{align*}
\]

(3) [4 points] Choose the incorrect statement, regarding the steady-state behavior of the illustrated circuit.

(a) There will be no voltage drop across the ideal inductor.
(b) Power will be dissipated only in the resistor R₂.
(c) There will be no voltage drop across \(\Delta V = \mathcal{E} \) for capacitor \( \neq 0 \).
(d) Current from the battery will equal \( \frac{\mathcal{E}}{R_2} \).
(e) There will be nonzero magnetic flux in L.

![Circuit Diagram]
(6) [12 points] A solid insulating sphere has radius $R_1$, and a uniform volume charge density $\rho$ of positive charge in it.

(a) Use Gauss’ Law to find the electric field at points both inside and outside the sphere. You must show the method here, indicating how the result is obtained.

Gauss' law, $\Phi_E = \frac{Q_{in}}{\epsilon_0}$, for a closed surface method: use gaussian spheres both inside & outside the sphere.

Inside: radius $r < R_1$ contains $Q_{in} = \rho \frac{4}{3} \pi r^3$

so $\Phi_E = E \times 4\pi r^2 = \frac{\rho}{3} \pi R_1^3 \epsilon_0$

or $E = \frac{\rho R_1^3}{3 \epsilon_0}$

Outside: $Q_{in} = \rho \frac{4}{3} \pi R_1^3$

$\Phi_E = E \times 4\pi R_1^2 = \frac{4\pi \rho R_1^3}{3 \epsilon_0}$

so $E = \frac{\rho R_1^3}{3 \epsilon_0 \cdot r^2}$

(b) Find a general relation for the electric potential at points outside the sphere, relative to infinity.

$\text{outside, } E \text{ acts like a } \frac{1}{r^2} \text{ point-charge field, }$

so $V = \frac{kQ}{r} \rightarrow V = \frac{\rho R_1^3}{3 \epsilon_0}$

$a, q. \ E = \frac{\rho R_1^3}{3 \epsilon_0 \cdot r^2} = \frac{\rho}{3} \pi r \epsilon_0 r^2 \rightarrow Q_{in} \text{ so } V = \frac{kQ_{in}}{r} = \frac{k \rho R_1^3}{r}$

same as above since $k = \frac{1}{4\pi \epsilon_0}$

(7) [7 points] In this circuit, the capacitor is initially uncharged. $V_B = 10 \text{ V, } C = 3.0 \mu F$, and $R = 3500 \Omega$. If the switch is closed at $t = 0$, find the voltage across the resistor at $t = 5.0 \text{ ms}$.

\[ V_R = V_B \times e^{-t/RC} \]

where $RC = 10.5 \text{ ms}$ in this case.

\[ V_R = (10V) e^{-5/10.5} = 10Ve^{-0.48} \]

\[ V_R = 6.2 \text{ V} \]

or note that $Q = Q_{max} \left(1 - e^{-t/RC}\right)$

\[ \frac{dQ}{dt} = C \frac{V_B}{R} e^{-t/RC} \]

\[ |I| = \frac{dQ}{dt} = C \frac{V_B}{R} e^{-t/RC} \]

\[ \frac{V_B}{R} e^{-t/RC} \]

so $V_R = \frac{V_B}{R} e^{-t/RC}$ as above.
(10) [7 points] A long, solid cylindrical wire carries a uniform current density $J$. Find the magnetic field inside the wire, far from the ends. For credit you must show how your result comes from a basic law. Which law did you use?

\[ \mathcal{J} = \frac{\mathcal{J}}{\pi R^2} \]

\[ \text{Solve is to use Ampere's law, } \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{in}}, \]

for a circular loop inside.

\[ I_{\text{in}} = \mathcal{J} \times (\text{loop area}) = \mathcal{J} \pi R^2 \]

\[ \oint \mathbf{B} \cdot d\mathbf{s} = B \times \text{circumference (since } B \text{ tangential)} \]

\[ = 2\pi R B \]

\[ \Rightarrow \quad 2\pi R B = \frac{\mu_0 J}{\pi} \pi R^2 \quad \Rightarrow \quad B = \frac{\mu_0 J R}{2} \]

(11) [9 points] Two long, parallel wires separated by a distance $d$ carry currents in opposite directions as shown in the figure. The top wire carries a current of 24 A. Point C is at the midpoint between the wires and point O is a distance 0.5d below the lower wire as in the figure. The total magnetic field at point O is zero.

(a) Find the current in the lower wire.

\[ I \]

\[ \bullet C \]

\[ \bullet O \]

\[ \text{at } O, \quad I \text{ gives } B \text{ down, } (\text{anticw } I) \rightarrow B \text{ up,} \]

by right hand rule

\[ \Rightarrow 0 = -\frac{\mu_0 I}{2\pi (3d/2)} + \frac{\mu_0 I'}{2\pi (d/2)} \]

\[ 2I' = \frac{2I}{3} \quad \Rightarrow \quad I' = I/3 \quad \text{or, } 8.0 \text{ A}. \]

(b) What is the direction of the magnetic field at C?

\[ \text{into page, both wires give } \vec{B} \text{ contributions the same direction.} \]
(12) [9 points] (a) If the capacitor value is \( C \), and the area is \( A \), find the electric field in the gap if the voltage \( V_1 \) is applied as shown. (Your answer should be in terms of \( C, A \), and \( V_1 \).) The shaded parts are metal plates; assume that these can be treated as very wide and flat.

\[
\begin{align*}
Q &= CV = \frac{\partial Q}{dV} = CV_1 \\
\sigma &= \frac{\partial Q}{A} = \frac{CV_1}{A} \quad \text{is & charge density on plates}
\end{align*}
\]

\[
E = \frac{\sigma}{\varepsilon_0} = \frac{CV_1}{\varepsilon_0 A}
\]

Another way: \( E = \frac{\Delta V}{d} = \frac{V_1}{d} \)

but in terms of \( \phi \), \( \varepsilon = \varepsilon_0 \frac{A}{d} \rightarrow d = \frac{\varepsilon_0 A}{\varepsilon} \)

\[
gives \quad E = \frac{V_1}{\Phi} \cdot \frac{\varepsilon}{\varepsilon_0 A} = \frac{CV_1}{\varepsilon_0 A} \quad \text{(the same)}
\]

(b) **Inside** either of the metal plates, which of the following will be true:

(i) The electric potential will be zero.
(ii) The electric field will be zero.
(iii) The electric field will be a constant vector pointing towards the center of the gap.
(iv) The charge density will be a constant value.
(v) The electric flux will be constant.
(8) [10 points] The resistors are arranged as shown.

(a) With $R_1$, $R_2$, and $R_3$ constants, find the net resistance of this circuit.

\[
\text{parallel } \Rightarrow \frac{R_2R_3}{R_2+R_3}
\]

in series with $R_1$ \Rightarrow \frac{R_1 + \frac{R_2R_3}{R_2+R_3}}{R_2 + R_3}

(b) If all three resistors each have $R = 30\Omega$, $V_a = 15\, V$, and $V_b = 60\, V$, find the power dissipated in $R_1$.

all $R$'s are same \Rightarrow R_{eff} = 30\, \Omega + 15\, \Omega = 45\, \Omega

so \quad I = \frac{\Delta V}{R_{eff}} = \text{through assembly, with } \Delta V = (60\, V - 15\, V) = 45\, V

\quad I = 1.0\, A \quad \text{in this case, all of which flows in } R_1 \Rightarrow P = I^2R_1 = 30\, W

(9) [7 points] Given a charged particle moving as shown,

(a) what must be the direction of the uniform magnetic field inside of the box, assuming there are no electric fields, and that gravity can be neglected?

\[
\begin{array}{c}
\text{i) in the plane of the paper, at an angle 45°} \\
\text{from the lower left to the upper right} \\
\text{ii) down into the paper} \\
\text{iii) to the right (–→)} \\
\text{iv) to the left (←)} \\
\text{v) up out of the paper} \\
\end{array}
\]

\[
Q \, \vec{v} \times \vec{B} \text{ is Force, (toward inside of curve)} \\
\rightarrow \text{B out of paper by r.h. rule,}
\]

(b) Along the path of the charge, which of the following is correct?

(i) The speed of the particle steadily increases.

(ii) The particle traces an arc of a circle.

(iii) The speed increases and decreases, oscillating at the cyclotron frequency.

(iv) The particle’s path is a parabola.

(v) The particle’s kinetic energy increases by $(B^2)/(2\mu_0)$, which is the energy density of the magnetic field in the box.
(4) [17 points] The figure shows three point charges, with \( Q \) a constant. The charges \((-Q)\) are located at \( \pm d \) on the \( x \) axis. The charge \(+4Q\) is located at position \((d/2)\) on the \( y \) axis.

(a) Find the force on the \((+4Q)\) charge, in terms of \( Q \) and \( d \). What is the direction?

\[
\text{there are two attractive } \vec{F} \text{ vectors, as shown, sum has no } x \text{-component,}
\]

\[
(\Sigma \vec{F})_y = -2 \left( \frac{k \cdot Q \cdot 4Q}{d^2 + (d/2)^2} \right) \cos \theta \cdot \frac{d}{\sqrt{2}}
\]

\[
= -\frac{8kQ^2}{(5d^2/4)} \cdot \frac{1}{\sqrt{5}} = -\frac{32kQ^2}{5\sqrt{5}d^2} \cos \theta \cdot \frac{d}{\sqrt{5}}
\]

\[
\text{or } -2.9 \frac{kQ^2}{d^2} \text{ in } y \text{-direction}
\]

(b) Find the electric potential at a point \((+d)\) on the \( y \) axis.

\[
\text{scalar sum of } 3 \text{ terms, } V = \sum \frac{kQ}{r_i} = 2 \cdot \frac{k(-Q)}{d\sqrt{2}} + \frac{k(+4Q)}{d/2}
\]

\[
= \frac{kQ}{d} (8 - \sqrt{2}) \text{ or } 6.6 \frac{kQ}{d}
\]

\(\text{(this is referenced to } \infty, V-V_\infty)\)

(c) Find the energy required to move the charge on the right to a point far away, if the other two charges remain fixed. Assume that \( Q = 2.5 \mu \text{C} \), and \( d = 4.0 \text{ cm} \) for this part.

\[
\Delta E = -\Delta U = -\sum \frac{kQ_iQ_2}{r_{12}} = -\frac{kQ^2}{(2d)} + \frac{k \cdot 4Q^2}{d \sqrt{5}/2} = \frac{kQ^2}{d} \left( \frac{8}{\sqrt{5}} - \frac{1}{2} \right)
\]

\[
\text{or } +3.1 \frac{kQ^2}{d} = +4.3 \text{ J}
\]

\(\text{(using } k = 9 \times 10^9 \text{ Nm}^2/\text{C}^2 \text{ and } d = 0.04 \text{ m})\)

(5) [5 points] An electromagnetic wave has an electric field with peak value 450 N/C. What is the average intensity of the wave?

\[
\bar{I} = \frac{\text{1m}}{2\mu_0} \text{ and } \vec{B} = \frac{\vec{E}}{c} \text{ gives}
\]

\[
\bar{I} = \frac{E_0^2}{2\mu_0 c} \quad \text{E}_0 = \text{peak value}
\]

\[
= \frac{(450 \text{N/C})^2}{2(4\pi \times 10^{-7} \text{Tm/A})(3 \times 10^8 \text{m/s})} = 270 \frac{\text{W}}{\text{m}^2}
\]