(1) Indicated in the center of the diagram is a long straight wire along the z axis, with current in the positive z direction. A uniform B field of magnitude 2.75 T is directed 45° from the x and y axes as shown (dashed lines).

(a) What is the direction of the force on the wire?

(b) Find the magnitude of the force on a length 3.0 m of the wire, if \( I = 3.5 \) A.

\[
\mathbf{F} = I \mathbf{l} \times \mathbf{B} \quad \Rightarrow \quad \mathbf{F} = l I B \sin \theta,
\]

\[
= (3.0 \text{ m}) \times 3.5 \text{ A} \times 2.75 \text{ T} \\
= 28.9 \text{ N} \approx 29 \text{ N}
\]

(2) Two long wires are shown in cross-section. The lower wire at position \((x=-2.0 \text{ cm}, y=-2.0 \text{ cm})\) carries \( I_1 = 6.0 \) A into the page, while the upper wire at \((-2.0 \text{ cm}, 2.0 \text{ cm})\) carries \( I_2 = 8.0 \) A out of the page.

(a) Find the magnetic field (magnitude and direction) at the origin.

\( \mathbf{B}_1 \) and \( \mathbf{B}_2 \) (due to \( I_1 \) and \( I_2 \)) are as shown - resultant points just above x axis

\[
\mathbf{B}_1 = \frac{\mu_0 I_1}{2\pi r} \hat{r} = 2\sqrt{2} \text{ cm} = 0.282 \text{ m}
\]

\[
= 4.2 \times 10^{-5} \text{ T} \quad \hat{z}
\]

\[
\mathbf{B}_2 = \frac{\mu_0 I_2}{2\pi r} \hat{r} = 4.8 \times 10^{-5} \text{ T} \quad \hat{y}
\]

Sum squares \( \Rightarrow \) \( \mathbf{B} = 7.1 \times 10^{-5} \text{ T} \)

\[
\cos \theta = \frac{8}{10} \Rightarrow \theta = 53.13^\circ \quad \text{so} \quad \theta = 8^\circ
\]

(b) Find the force (with direction) on a 2.0 m length of \( I_1 \), due to the current \( I_2 \).

due to \( I_1 \) \( \mathbf{B}_1 \) at \( I_2 \) points right \( \Rightarrow \) \( \mathbf{F}_2 \) points up

\[
\mathbf{F} = \frac{\mu_0 I_1 I_2 \mathbf{L}}{2\pi r} \quad \Rightarrow \quad \mathbf{F}_2 \text{ points up}
\]

\[
= 4.8 \times 10^{-4} \text{ N}
\]

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(3) A very long solenoid has \( n \) turns per unit length, radius \( R \), and carries a current \( I \).

(a) Use Ampere's law to deduce the magnetic field inside this solenoid. You must show how you obtain the field to get credit.

\[
\oint B \cdot dl = \mu_0 I_{\text{enc}} \quad \text{Ampere's law}
\]

Choose a loop like this -

\[
\oint B \cdot dl = \mu_0 I_{\text{enc}}
\]

\[
B \cdot dl = B d \text{ since only the horizontal leg inside contributes.}
\]

So,

\[
B d = \mu_0 n d I
\]

\[
B = \mu_0 n I.
\]

(b) For a length \( d \) of the solenoid, what is the energy stored inside, in terms of \( I, R, n, \) and \( d \)?

\[
\text{Use } u = \frac{B^2}{2\mu_0} \text{ is energy/} \text{vol.}
\]

So,

\[
U = u \cdot \text{vol} = \frac{B^2}{2\mu_0} \cdot (\pi R^2 d)
\]

\[
= \left(\frac{n\mu_0 I}{\mu_0} \right)^2 \pi R^2 d
\]

\[
= \frac{\mu_0 n^2 I^2 \pi R^2 d}{2}
\]

(4) For a long solenoid such as described in the problem above, suppose the current is increasing with time. Which of the following is a true statement regarding this situation? (Choose one.)

(i) The increase in current will be balanced by a decrease in the displacement current, giving no change in the magnetic field inside the solenoid.

(ii) The electric field will be zero; only a magnetic field will be produced.

(iii) The electric field will point along the axis of the solenoid, parallel to the magnetic field lines.

(iv) Lenz's law tells us that the increasing current will cause a corresponding decrease in the energy stored in the solenoid.

(v) Faraday's law shows that there will be electric fields that circulate within the solenoid.
In the circuit shown, the inductor and battery are ideal (no resistance), and the switch is opened at time $t = 0$, after having been closed for a long time.

(a) What energy is stored in the inductor before the switch is opened, if $V_B = 12.0 \, \text{V}$, $R = 5.0 \, \Omega$, $L = 3.0 \, \text{mH}$, and $C = 1200 \, \mu\text{F}$?

We need, $I = \frac{V_B}{R}$ in this case

So $U = \frac{1}{2} LI^2 = \frac{L}{2} \left( \frac{V_B}{R} \right)^2 = \frac{0.003 \, \text{H} \cdot (12 \, \text{V})^2}{2} = 8.6 \times 10^{-3} \, \text{J}$

(b) Just before the switch is opened, what is the emf in the inductor?

$\varepsilon_L = 0$ since $\frac{di}{dt} = 0$ at that time

(c) After opening the switch, what is the peak value of energy that will be stored in the capacitor at later times? (Use the values given in part (a).

We know energy alternates between $L \& C$ in this case $\Rightarrow$ peak value in $C$ same as peak (initial) value in $L$

$\Rightarrow U = 8.6 \times 10^{-3} \, \text{J}$

(d) Find the current in the inductor at time $t = 4.0 \, \text{ms}$ (Use the values given in part (a).

$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{(0.003 \, \text{H} \cdot 0.0012 \, \text{F})^{1/2}} = 527 \, \text{s}^{-1}$

and at $t = 4 \, \text{ms}$, $\omega t = 2.11 \, \text{(radians)}$

$\Rightarrow I = I_0 \cos \omega t$ where $I_0 = \frac{V_B}{R}$ as in (a)

$I = (2.4 \, \text{A}) \cos \omega t = (-1.2 \, \text{A})$
(6) A square loop abcd has dimensions $d$ by $d$, and is moving with velocity $v$ to the right (along positive $x$). At the instant $t = 0$, the left side of the loop aligns with the $y$ axis, as in the picture. This loop feels a field $B$ directed perpendicular to the plane, however $B$ increases linearly with $x$ as follows: $B = B_0 \left(1 + \frac{x}{d}\right)$, where $d$ is a constant length (same as the loop dimension), and $B_0$ is a constant field.

(a) What emf will develop across each of the four segments $ab$, $bc$, etc., at some time $t > 0$, where the loop is completely inside the region of changing $B$?

(b) What is the total emf around the loop? Does it go clockwise or counterclockwise (give a brief explanation)?

(c) Determine the current in the loop as it moves along, if $v = 9.0 \text{ m/s}$, $d = 1.9 \text{ cm}$, $B_0 = 11.0 \text{ T}$, and the 1-turn loop has resistance $R = 19 \Omega$. Does the current change with time?

(d) Determine the direction of the force on each of the four segments of the square loop (values not needed), and the direction of the net force on the loop.