Introduction

The purpose of this lab is to help you understand velocity and acceleration. These are vector quantities but in this lab you will be dealing with one dimensional motion and will thus avoid the vector complexity. These experiments and the analysis of the data will give you a concrete example of the limiting process you are learning about in calculus.

The Apparatus

You will have a track and a cart that can roll, almost without friction, along the track. The key to the experiment is a position sensor that can measure and record the position of a point on the cart for a large number of times that are extremely close together. The time interval between measurements is fixed by the apparatus. From this data which you should call $x(t)$ you will be able to calculate the velocity as a function of time and the acceleration, which should be constant. You should make two runs with one end of the track elevated, at different heights, so that the cart accelerates. Knowing the angle that the track makes with the horizontal tells you what the acceleration should be and this theoretical number can be compared to the experimental number you will obtain from your data.

Procedure

Your lab instructor will show you how to set up the motion detector and computer so that you can take and record measurements of the cart’s position. Once you know how to do this you are to carry out two experiments. For each one you should obtain a large number of values of $x(t)$ which you need to store along with the values of $t$ in an Excel spreadsheet so that you can later analyze the data. The steps you should follow are:

1. To record the data you click on the Logger Pro icon on your computer. You just hit the Collect button on the screen and then release the cart and the data is automatically recorded. (The apparatus only works if the cart is at least 40cm from the sensor.)
2. Go to the **Window** menu, choose **Table** and select **tall**. Determine from the graph the time when the motion stopped. In the table select the time and distance columns down to the point where the motion stopped. Copy this information by using **Control c**.

3. Open **EXCEL** and paste using **Control v** the data into the Excel spreadsheet.

4. Choose the **CHART WIZARD** which looks like a bar graph. Select **XY** scatter plot. For the **range** choose the distance data, which should be in the second column. Select the **SERIES** tab and put in the time, which should be in the first column, as the horizontal values (here called by Excel x even though its the time!) Then hit **FINISH**.

5. Click on the graph and then select the **Chart** menu. Go to **Trendlines**. For the **Type** select **Polynomial** and for **Order** put in 2. Click on **Options** and select **Display Equation on Chart**. You should now have the function \( x(t) \) that has been the object of so much attention in the lectures.

**Analysis of Data**

Given the polynomial fit to the data you should calculate the derivative of \( x(t) \) using the rules for differentiation. You then have the velocity as a function of time. The goal is to verify that you can get the velocity at any particular time by doing the limit process. Choose a time early in the range of \( t \) and call it \( t_1 \). We want \( t_2 \) to be a later time at which \( x(t) \) was recorded. Therefore let

\[
 t_2 = t_1 + \Delta t.
\]

There was a certain time interval between data points determined by the apparatus. Let us call this time interval \( z \). Then we can write

\[
 \Delta t = nz
\]

where \( n \) is the number of intervals between \( t_1 \) and \( t_2 \). (Now by changing the integer \( n \) we can change the size of \( \Delta t \).) From the column of \( x(t) \) in the spread sheet, calculate

\[
\frac{x(t_2) - x(t_1)}{t_2 - t_1}
\]

with

\[
 t_2 = t_1 + nz
\]

for \( n \) decreasing from some large number to 1. What you are calculating is

\[
\frac{x(t_1 + nz) - x(t_1)}{t_1 + nz - t_1} = \frac{x(t_1 + nz) - x(t_1)}{nz}
\]
and the limit as $n$ goes to a small number is the limit as $\Delta t$ goes to zero. Plot the results as a function of $n$. If everything works as it should, in the limit of $\Delta t$ approaching zero the value for $v(t_1)$ should be the same as what you get by putting $t = t_1$ in the function $v(t)$ obtained by taking the derivative of $x(t)$. You might repeat this process for a few values of $t_1$.

Repeat this process for the other set of data with the different elevation of the track.

Checking the Acceleration

The component of the acceleration along the track should be $a_x = g \sin \theta$ where $\theta$ is the angle that the track makes with the horizontal. Using the polynomial fit and differentiating twice you should calculate the experimental value of the acceleration. You should compare the theoretical and experimental values to see that they are approximately the same.