Dynamics of Rotational motion

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Angular Momentum

First way to define the Angular Momentum $\vec{L}$:

$$\vec{L} = I \vec{\omega}$$

$$\sum \vec{\tau} = I \vec{\alpha} = I \frac{d\vec{\omega}}{dt} = \frac{d(I\vec{\omega})}{dt} = \frac{d(\vec{L})}{dt} = \frac{d\vec{L}}{dt}$$

$$\sum \vec{\tau} = I \vec{\alpha} = \frac{d\vec{L}}{dt}$$
Angular Momentum Definition

Another definition:

\[ \vec{L} = \vec{r} \times \vec{p} \]
Angular Motion of a Particle

• Determine the angular momentum, $L$, of a particle, with mass $m$ and speed $v$, moving in circular motion with radius $r$.
Momentum

Momentum vs. Angular Momentum:

\[ \vec{p} = m \vec{v} \rightarrow \vec{L} = I \vec{\omega} \]

Newton's Laws:

\[ \vec{F} = \frac{d\vec{p}}{dt} \rightarrow \vec{\tau} = \frac{d\vec{L}}{dt} \]
Conservation of Angular Momentum

\[ \sum \vec{\tau} = \frac{d\vec{L}}{dt} \]

if \[ \sum \tau = 0 \rightarrow L = \text{Const} \]

By Newton's laws, the angular momentum of a body can change, but the angular momentum for a system cannot change.

Conservation of Angular Momentum

Same as for linear momentum
Ice Skater

- Try this at home in a chair that rotates
- Get yourself spinning with your arms and legs stretched out, then pull them in

\[ \vec{L} = I \vec{\omega} \]

- \( I \) large, \( \omega \) small
- \( I \) small, \( \omega \) large

(a) (b)
Problem Solving

• For Conservation of Angular Momentum problems:

BEFORE and AFTER
Clutch Design

- As a car engineer, you model a car clutch as two plates, each with radius $R$, and masses $M_A$ and $M_B$ ($I_{\text{Plate}} = \frac{1}{2}MR^2$).
- Assuming plate $A$ spins with speed $\omega_1$ and Plate $B$ is at rest, you want to engage them so they spin together.
- Find the final angular velocity of the system.
Rotational Kinetic Energy

\[ KE_{\text{trans}} = \frac{1}{2} mv^2 \]

\[ KE_{\text{rotate}} = \frac{1}{2} I \omega^2 \]

Conservation of Energy must take into account rotational kinetic energy.
Rotation and Translation

- Objects can both **Rotate** and **Translate**
- Need to add the two

$$KE_{total} = \frac{1}{2} mv^2 + \frac{1}{2} I \omega^2$$

- Rotational part: pick axis going through the **CM**
- Rolling without slipping is a special case where you can relate the two
  - $$v = \omega r$$

Reminder
Rolling Down an Incline

- You take a solid ball of mass $m$ and radius $R$ and hold it at rest on a plane with height $Z$. You then let go and the ball rolls without slipping.
- What will be the speed of the ball at the bottom?
- What would be the speed if the ball didn’t roll and there were no friction? Larger or smaller? What is physical meaning?

Note: $I_{\text{sphere}} = \frac{2}{5}mR^2$
A bullet strikes a cylinder

- A bullet of speed $V$ and mass $m$ strikes a solid cylinder of mass $M$ and inertia $I = \frac{1}{2}MR^2$, at radius $R$ and sticks. The cylinder is anchored at point $O$ and is initially at rest.

- What is $\omega$ of the system after the collision?

- Is energy conserved?
Yo-Yo

- Three identical yo-yos are initially at rest. You pull on the string as shown below. There is enough friction for yo-yo to roll without slipping in each case.

- In which direction will each of the yo-yos rotate?
Coming up…

• Chapter 11 - read it!

Hints: Reading questions due: Q11.1-11.4 & Q11.8-11.11, Q11.15