Physics 218: sect.513-517

Dynamics of Rotational motion

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Torque

• Torque is our “slamming” ability

• Write Torque as $\tau$

\[
\tau = |r| |F| \sin \theta
\]

\[
\vec{\tau} = \vec{r} \times \vec{F}
\]

• To find the direction of the torque, wrap your fingers in the direction the torque makes the object twist
We discussed how to find the torque and what is expected from you when you are solving problems with torques.
Torque and Moment of Inertia

• Force vs. Torque

\[ F = ma \quad \Rightarrow \quad \tau = I \alpha \]

• Mass vs. Moment of Inertia

\[ m \Rightarrow I = \sum m r^2 \quad \text{or} \quad I = \int r^2 \, dm \]
Rotational Kinetic Energy

\[ KE_{\text{trans}} = \frac{1}{2}mv^2 \]

\[ KE_{\text{rotate}} = \frac{1}{2}I\omega^2 \]

Conservation of Energy must take into account rotational kinetic energy
Rotation and Translation

- Objects can both **Rotate** and **Translate**
- Need to add the two

\[ KE_{total} = \frac{1}{2} \, mv^2 + \frac{1}{2} \, I \omega^2 \]

- Rotational part: pick axis going through the CM
- Rolling without slipping is a special case where you can relate the two
  - \( V = \omega r \)
Heavy Pulley

- A heavy pulley, with radius $R$, starts at rest. We pull on an attached rope with a constant force $F_T$. It accelerates to final angular speed $\omega_f$ in time $t_f$.
- What is the moment of Inertia?
More Realistic Heavy Pulley

- A heavy pulley, with radius $R$, starts at rest. We pull on an attached rope with constant force $F_T$. It accelerates to final angular speed $\omega$ in time $t$.
- A better estimate takes into account that there is friction in the system. This gives a torque (due to the axel) we'll call this $\tau_{\text{fric}}$.
- What is this better estimate of the moment of Inertia?
Pulley and Bucket

• A heavy pulley, with radius $R$, and known moment of inertia $I$ starts at rest. We attach it to a bucket with mass $m$. The friction torque is $\tau_{\text{fric}}$.

• Find the angular acceleration $\alpha$
Angular Quantities

- Position $\rightarrow$ Angle $\theta$
- Velocity $\rightarrow$ Angular Velocity $\omega$
- Acceleration $\rightarrow$ Angular Acceleration $\alpha$
- Force $\rightarrow$ Torque $\tau$
- Mass $\rightarrow$ Moment of Inertia $I \omega^2/2$
- Energy $\rightarrow$ Rotational Energy $I$

Today we'll finish:
- Momentum
Momentum

Momentum vs. Angular Momentum:

\[ \vec{p} = m\vec{v} \quad \rightarrow \quad \vec{L} = I\vec{\omega} \]

Newton's Laws:

\[ \vec{F} = \frac{d\vec{p}}{dt} \quad \rightarrow \quad \vec{\tau} = \frac{d\vec{L}}{dt} \]
Angular Momentum

First way to define the Angular Momentum $\mathbf{L}$:

\[
\sum \mathbf{\tau} = I \mathbf{\ddot{\alpha}} = I \frac{d\mathbf{\ddot{\omega}}}{dt} = \frac{d(I\mathbf{\ddot{\omega}})}{dt} = \frac{d(\mathbf{L})}{dt} = \frac{d\mathbf{L}}{dt}
\]
Angular Momentum Definition

Another definition:

\[ \vec{L} = \vec{r} \times \vec{p} \]
Angular Motion of a Particle

• Determine the angular momentum, $L$, of a particle, with mass $m$ and speed $v$, moving in circular motion with radius $r$.

To be continued
Coming up…

• Chapter 11 (and finish 10!) - read it!

Hints: Reading questions due: Q10.13-10.15 & Q10.17-10.20, Q10.26

Ch. 11 questions will be posted online (in this lectures notes)