Dynamics of Rotational motion

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Overview: Rotational Motion

- Take our results from "linear" physics and do the same for "angular" physics
- Analogue of
  - Position ←
  - Velocity ←
  - Acceleration ←
  - Force ←
  - Mass ←
  - Momentum ←
  - Energy ←

Chapters 1-3

Chapters 4-7
Angular Quantities

• So far:
  - Position $\rightarrow$ Angle $\theta$
  - Velocity $\rightarrow$ Angular Velocity $\omega$
  - Acceleration $\rightarrow$ Angular Acceleration $\alpha$

• Now, we’ll start discussing the vector nature of the variables and then move forward on the others:
  - Force
  - Mass
  - Momentum
  - Energy
Angular Quantities

- Position $\rightarrow$ Angle $\theta$
- Velocity $\rightarrow$ Angular Velocity $\omega$
- Acceleration $\rightarrow$ Angular Acceleration $\alpha$

Moving forward:
- Force
- Mass
- Momentum
- Energy
Slamming a door

We know this from experience:

- If we want to slam a door really hard, we grab it at the end.
- If we try to push in the middle, we aren’t able to make it slam nearly as hard.
Torque

• **Torque** is the analogue of **Force**

• Take into account the perpendicular distance from axis

  - Same force further from axis leads to more Torque

\[ \text{Torque} = \text{Force} \times \text{Distance from axis} \]

![Diagram showing two forces, \( F_1 \) and \( F_2 \), at different distances from the axis, \( R_1 \) and \( R_2 \).]
Slamming a door

We also know this from experience:

- If we want to slam a door really hard, we grab it at the end and “throw” perpendicular to the hinges.
- If we try to pushing towards the hinges, the door won’t even close.
• What if we change the angle at which the Force is applied?

• What is the “Effective Radius?”
Torque

- Torque is our “slamming” ability
- Write Torque as $\tau$

$$|\tau| = r \| F \| \sin \theta$$
Torque and Force

Torque problems are like Force problems

1. Draw a force diagram
2. Then, sum up all the torques to find the total torque

Is torque a vector?
Torque

• Torque is our “slamming” ability
• Write Torque as $\tau$

$$|\tau| = r \| F \| \sin \theta$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

• To find the direction of the torque, wrap your fingers in the direction the torque makes the object twist
Torque and Force

Torque problems are like Force problems
1. Draw a force diagram
2. Then, sum up all the torques to find the total torque

Is torque a vector?
Vector Cross Product

\[ \vec{C} = \vec{A} \times \vec{B} \]
\[ |C| = |A| |B| \sin \theta \]

This is the last way of multiplying vectors we will see

- Direction from the “right-hand rule”
- Swing from A into B!
Vector Cross Product Cont...

- Multiply out, but use the $\sin \theta$ to give the magnitude, and RHR to give the direction.

\[
\hat{i} \times \hat{i} = 0 \quad (\sin \theta = 0)
\]
\[
\hat{i} \times \hat{j} = \hat{k} \quad (\sin \theta = 1)
\]
\[
\hat{i} \times \hat{k} = -\hat{j} \quad (\sin \theta = 1)
\]
Example: Composite Wheel

- Two forces, $F_1$ and $F_2$, act on different radii of a wheel, $R_1$ and $R_2$, at different angles $\Theta_1$ and $\Theta_2$. $\Theta_1$ is a right angle.

- If the axis is fixed, what is the net Torque on the wheel?
Angular Quantities

- Position $\Rightarrow$ Angle $\theta$
- Velocity $\Rightarrow$ Angular Velocity $\omega$
- Acceleration $\Rightarrow$ Angular Acceleration $\alpha$

Moving forward:

- Force $\Rightarrow$ Torque $\tau$
- Mass
- Momentum
- Energy
Analogue of Mass

The analogue of Mass is called *Moment of Inertia*

Example: A ball of mass $m$ moving in a circle of radius $R$ around a point has a moment of inertia

$$F=ma \Rightarrow \tau = I \alpha$$
Calculate Moment of Inertia

• Calculate the moment of inertia for a ball of mass $m$ relative to the center of the circle $R$
Moment of Inertia

• To find the mass of an object, just add up all the little pieces of mass

• To find the moment of inertia around a point, just add up all the little moments (each is \(mr^2\))

\[ I = \sum mr^2 \quad \text{or} \quad I = \int r^2 \, dm \]
Torque and Moment of Inertia

• Force vs. Torque

\[ F = ma \Rightarrow \tau = I \alpha \]

• Mass vs. Moment of Inertia

\[ m \Rightarrow I = \sum mr^2 \quad \text{or} \quad I = \int r^2 \, dm \]
Moment of Inertia for a Disc

• Calculate moment of inertia for a thin disk of mass $M$ and radius $R$

• How will the result change if this is a cylinder of length $l$ (i.e. a very thick disc)?
Rotational Kinetic Energy

\[ KE_{\text{trans}} = \frac{1}{2}mv^2 \]

\[ KE_{\text{rotate}} = \frac{1}{2}I\omega^2 \]

Conservation of Energy must take into account rotational kinetic energy
Rotation and Translation

- Objects can both Rotate and Translate
- Need to add the two

\[ KE_{total} = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 \]

- Rotational part: pick axis going through the CM
- Rolling without slipping is a special case where you can relate the two
  - \( V = \omega r \)
Next Time

• The rest of Chapter 10
  - More on “angular stuff”
  - Angular Momentum
  - Energy
Coming up...

• Next week:
  - Homework 8 - BOTH PARTS! Was due today

• Chapter 10 (and 9!) - read it!

  Hints: Reading questions due: Q10.13-10.15 & Q10.17-10.20, Q10.26