Physics 218: sect.513-517

Lecture 5: Kinematics (continued)

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iClickers

• Don’t forget about batteries. They die. (have a spare set)
Important Equations of Motion

If the acceleration is constant

\[ \vec{v} = \vec{v}_0 + \vec{a}t \]

\[ \vec{x} = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a}t^2 \]

Position, velocity and Acceleration are vectors. More on this in Chap 3
Getting Displacement from Velocity

For const acceleration the

Equation of motion: \( X = X_0 + V_0 t + \frac{1}{2}at^2 \)

- If you are given the velocity vs. time graph you can find the total distance traveled from the area under the curve:

\[
X - X_0 = V_0t + \frac{1}{2}at^2
\]

- Can also find this from integrating...

\[
x - x_0 = \int_0^t v \, dt
\]
Definite and Indefinite Integrals

How do you calculate the Value of an integral?

In many ways an integral is an anti-derivative

\[ \Rightarrow \int (c) \, dt = ct + b \]

where \( b \) is an arbitrary constant and is added to the right side of the equation

\[ \Rightarrow \frac{d(ct + b)}{dt} = c \]

If I know where my region of integration begins and ends:

(assuming \( a, b, \) and \( c \) are constants)

\[ \int_{a}^{b} c \, dt = ct \bigg|_{t=a}^{t=b} = cb - ca \]
Some Integrals

Make this more general:

\[ \int a \, dt = at + c \]

\[ \int at^m \, dt = a\left( \frac{t^{m+1}}{m+1} \right) + c \]

Check "anti-derivative"

\[ \frac{d}{dt}\left( \int at^m \, dt \right) = \frac{d}{dt}\left( a\left( \frac{t^{m+1}}{m+1} \right) + c \right) \]

\[ = (m + 1)a\left( \frac{t^{(m+1)-1}}{m+1} \right) + 0 \]

\[ = at^m \]
Our Example with Const. Acceleration

\[ x - x_0 = \int_0^t v \, dt \]

\[ = \int_0^t (v_0 + at) \, dt \]

\[ = \left[ (v_0 t + \frac{1}{2} at^2) \right]_0^t \]

\[ = v_0 t + \frac{1}{2} at^2 \]
How quickly can you stop a car?

You’re driving along a road at some constant speed, $V_0$, and slam on the breaks and slow down with constant deceleration $a$.

1. How much time does it take to stop?
2. How far do you travel before you come to a stop?

When you hit the brakes

Where you stop
How to solve

1. Draw the velocity and acceleration vectors.
2. Use the equations:
   - $\mathbf{x}(t) = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2} \mathbf{a} t^2$
   - $\mathbf{v}(t) = \mathbf{v}_0 + \mathbf{a} t$
3. Project on the $X$ axis.
4. Use the equations:
   - $x(t) = x_0 + v_0 t - \frac{1}{2} a t^2$
   - $v(t) = v_0 - at$
5. Additional conditions:
   - $u(t_f) = 0$
   - $u(t) = \frac{dx}{dt} = v_0 - at$
6. Solve:
   - $v(t_f) = v_0 - at_f = 0 \Rightarrow t_f = \frac{v_0}{a}$

$y = ax^2 + bx + c = 0, \quad -b \pm \sqrt{b^2 - 4ac}$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- **7:** Check if the result is reasonable
Problem

• Show that for constant acceleration:

$$2a(\Delta x) = V_f^2 - V_0^2$$

• Derive it at home!
• Be careful applying this formula!
• It is likely to be used in the exam
Free Fall

• Free fall is a good example for one-dimensional problems

• Gravity:
  - Things accelerate towards earth with a constant acceleration
  - $a = g = 9.8 \text{m/s}^2$ towards the earth
  - We’ll come back to Gravity a lot!
Throw a Ball up

You throw a ball upward into the air with initial velocity $V_0$. Calculate:

a) The time it takes to reach its highest point (the top).

b) Distance from your hand to the top

c) Time to go from your hand and come back to your hand

d) Velocity when it reaches your hand

e) Time from leaving your hand to reach some random height $h$. 
Throw a Ball up (solution)

• Finish solving the last part (quadratic equation)
• Analyze for which values of “h” it has solutions
Quadratic equation

$$\Delta = b^2 - 4ac.$$
Graphs

• Describe motion in each point:
  - Direction
  - Velocity
  - Acceleration

• Be prepared for this type of problems
Wednesday & later

• Reading and Lecture: Chapter 3, *Motion in two or three dimensions*

• Reading Questions: Discussion Questions Q3.5 & Q3.6, due at the beginning of class (these questions are hints for the clicker quiz)

• Recitation and Homework:
  - **HW:** Chapter 2 HW is due Monday 8 am
  - **Recitation:** Chapter 3 and Quiz
  - **Recitation Prep:** Do HW3 before recitation